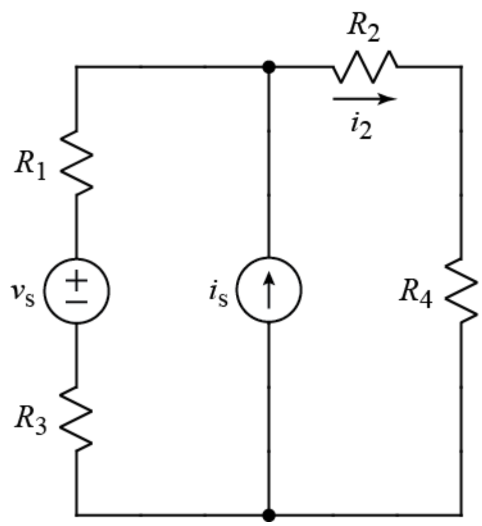
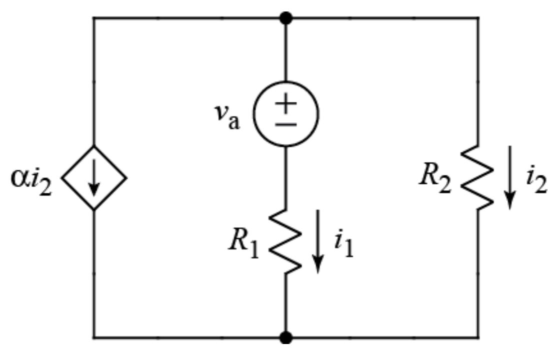
HOMEWORK #5 prob 2 solution

Ex:



a) Derive an expression for i_2. The expression must not contain more than the circuit parameters v_s, i_s R_1, R_2, R_3 and R_4.

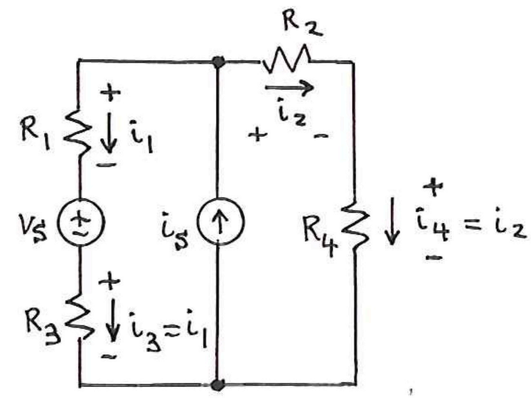
b) Derive an expressions for the power dissipated by resistor R_4. The expression must not contain more than the circuit parameters v_s, i_s R_1, R_2, R_3 and R_4.



c) Derive an expression for i_1. The expression must not contain more than the circuit parameters v_a, R_1, R_2, and α. **Note:** $\alpha > 0$.

SOL'N: a)

Since there is no simple solution such as a V-divider or i-divider, we use Kirchhoff's and Ohm's laws.



In the circuit diagram, only i's are shown, with the understanding that v's are given by Ohm's law: $v = iR$.

We have only one v-loop, (that avoids i-src's), around the outside of the circuit.

$$\underset{\underset{i_1}{\parallel}}{i_3} R_3 + v_s + i_1 R_1 - i_2 R_2 - \underset{\underset{i_2}{\parallel}}{i_4} R_4 = 0\,V$$

For current sums, we first look for R's in series that have the same current. This gives $i_1 = i_3$ and $i_2 = i_4$.

Next we sum currents at the top (or bottom) node. Note that we only use one node, as the other node is redundant. Currents measured flowing into the top node total to the same value as currents measured flowing out of the top node.

$$i_s = i_1 + i_2$$

We eliminate i_1 and then solve for i_2.

$$i_1 = i_s - i_2$$

substitute into the v-loop eq'n:

$$(i_s - i_2)(R_3 + R_1) + v_s - i_2(R_2 + R_4) = 0\,\text{V}$$

or

$$-i_2(R_1 + R_2 + R_3 + R_4) + i_s(R_1 + R_3) + v_s = 0\,\text{V}$$

or

$$i_2 = \frac{i_s(R_1 + R_3) + v_s}{R_1 + R_2 + R_3 + R_4}$$

b)

Power $p = i^2 R$. R_4 is in series with R_2, so $i_4 = i_2$.

$$p = i_2^2 R_4 = \left(\frac{i_s(R_1 + R_3) + v_s}{R_1 + R_2 + R_3 + R_4}\right)^2 R_4$$

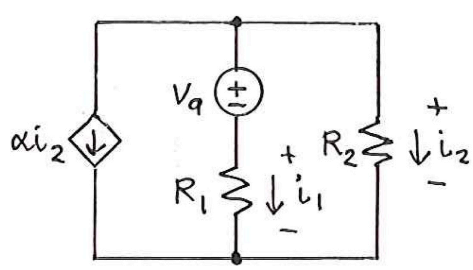
c)

We use Kirchhoff's laws.

We have no R's in series having the same i, so we move on to a current sum at the top node.

$$\alpha i_2 + i_1 + i_2 = 0\,\text{A}$$

We have one v-loop that doesn't pass thru a current source. It is on the right.



The voltage drops are $v_1 = i_1 R_1$ and $v_2 = i_2 R_2$.

$$i_1 R_1 + v_a - i_2 R_2 = 0\,V$$

The current sum for the top node gives a second equation.

$$\alpha i_2 + i_1 + i_2 = 0\,A$$

We eliminate i_2 by using the second eq'n and substituting for i_2 in the first eq'n.

$$i_2 (1 + \alpha) = -i_1$$

or

$$i_2 = \frac{-i_1}{1+\alpha}$$

Substituting into the first eq'n, we have

$$i_1 R_1 + V_a - \frac{-i_1}{1+\alpha} R_2 = 0\,V$$

or

$$i_1 \left(R_1 + \frac{R_2}{1+\alpha} \right) = -V_a$$

or

$$i_1 = \frac{-V_a}{R_1 + \frac{R_2}{1+\alpha}}$$