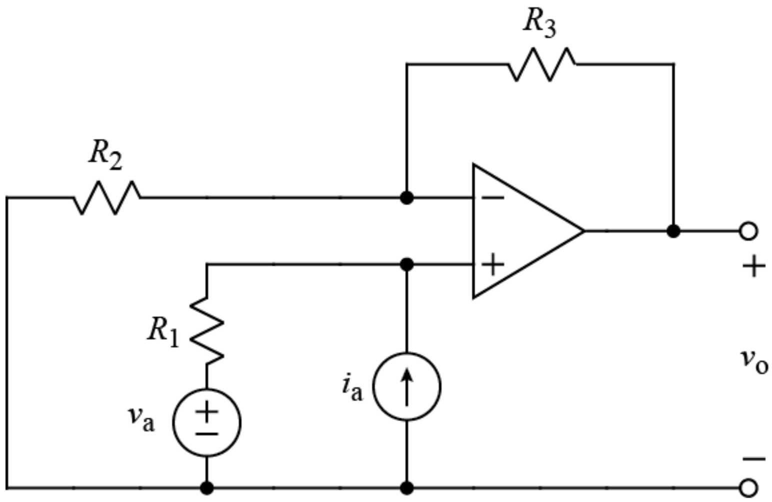
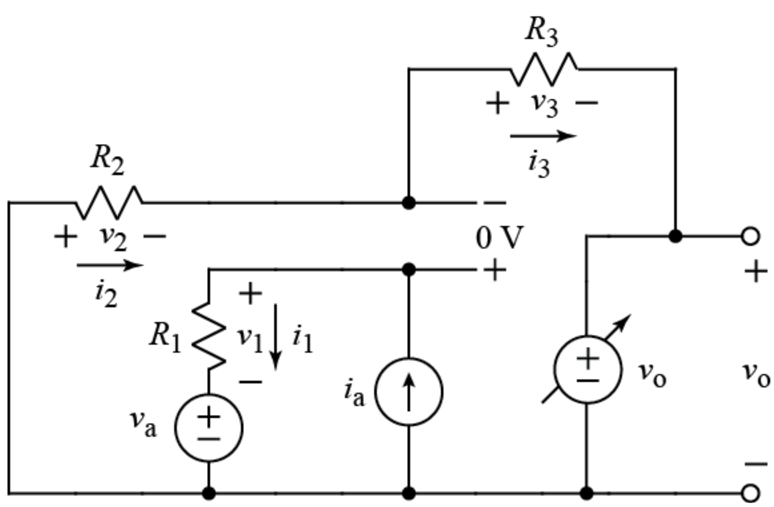
Ex:



The op-amp operates in the linear mode. Using an appropriate model of the op-amp, derive an expression for v_o in terms of not more than v_a, i_a, R_1, R_2, and R_3.

SOL'N: For a solution using Kirchhoff's and Ohm's laws, erase the op-amp and replace it with a 0 V drop across the + and – inputs and a source called v_0 for the output. The next step is to apply voltage and current measurement labels to each resistor. One way this labeling can be done is shown in the figure below.



From this point on, writing the v-loop, i-sum, and Ohm's law equations yields the information needed to find the formula for v_0

v-loops (being careful to ensure to include a loop passing through the 0 V drop across the op-amp inputs):

left-side loop: $-v_2 - v_1 - v_a = 0\,\text{V}$

right loop: $v_a + v_1 - 0\,\text{V} - v_3 - v_o = 0\,\text{V}$

Note: These loops avoid going through the current source. Also, the outer loop is redundant and so is omitted.

i-sums (at nodes not connected to other nodes by only v-sources):

– input i-sum: $i_2 = i_3$

+ input i-sum: $i_a = i_1$

Note: The other nodes are at v_o and the bottom "rail", but these are connected to each other by only a v-source.

Ohm's law:

$v_1 = i_1 R_1$

$v_2 = i_2 R_2$

$v_3 = i_3 R_3$

Now the algebra. There are seven equations in seven unknowns. Starting with the i-sum equations first, since they are the simplest, we substitute i_2 for i_3 and i_a for i_1 in the Ohm's law equations.

$v_1 = i_a R_1$

$v_2 = i_2 R_2$

$v_3 = i_2 R_3$

At this point, we have used the i-sum equations and no longer consider them. Next, since the Ohm's law equations are the simplest equations that remain, we use them to substitute for v's in the v-loop equations.

left-side loop: $-i_2 R_2 - i_a R_1 - v_a = 0\,\text{V}$

right loop: $v_a + i_a R_1 - i_2 R_3 - v_o = 0\,\text{V}$

The left-side loop equation contains sufficient information to determine the value of i_2 in terms of sources and resistances.

$$i_2 = -\frac{i_a R_1 + v_a}{R_2}$$

Finally, substituting for i_2 in the right loop yields an expression for v_o in terms of sources and resistors.

$$v_{\mathrm{o}} = v_{\mathrm{a}} + i_{\mathrm{a}} R_1 - \left(-\frac{i_{\mathrm{a}} R_1 + v_{\mathrm{a}}}{R_2} \right) R_3$$

or

$$v_{\mathrm{o}} = \left(v_{\mathrm{a}} + i_{\mathrm{a}} R_1 \right) \left(1 + \frac{R_3}{R_2} \right)$$