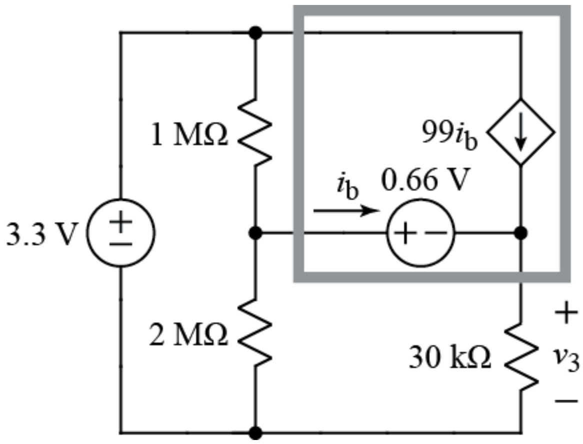
Ex:

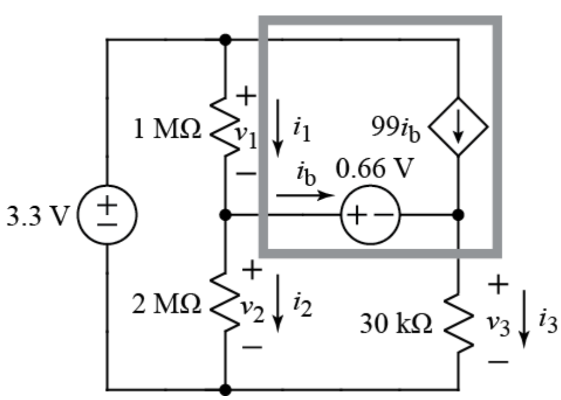


Find i_b, v_3, and the power dissipated by the components in the box.

SOL'N: The power for the box is given by the sum of the powers for the components inside the box.

$$p = \sum_{parts} iv$$

Kirchhoff's laws and Ohm's law will yield the values of currents and voltages in the circuits. Thus, the first step is to label voltage drops and currents for all the resistors. Either direction for a voltage drop is acceptable in the absence of a specified direction so long as the labeling of the current measurement obeys the passive sign convention (with an arrow pointing toward the – sign of the v-drop). Thus, we have two choices for the direction of the voltage measurement across each resistor. The (arbitrary) choice made here is to put the + sign on top for both resistors.



The voltage loops on the left and lower right avoid passing through the current source and yield the following equations when proceeding in a clockwise direction (and using the v-drop sign where we exit each component):

$$3.3\text{V} - v_1 - v_2 = 0\text{V} \tag{1}$$

$$v_2 - 0.66\text{V} - v_3 = 0\text{V} \tag{2}$$

These are the only v-loops required. Taking a larger v-loop through the 3.3 V source, 1 MΩ resistor, 0.66 V source, and 30 kΩ resistor is redundant. That v-loop equation turns out to be the sum of (1) and (2).

Turning to current summations at nodes, the top node is ignored because we do not have a defined current measurement for the 3.3 V source on the left. The same applies to the node on the bottom. Thus, only the node in the center and the node on the right side are candidates for current summations. Usually, the 0.66 V source between nodes would lack a current measurement, but here the current i_b is defined. It happens that i_b must be used in at least one current summation since it is the current upon which the dependent source, $99i_b$, depends.

For the middle and right nodes, the following current summation equations are found (from measured in = measured out):

$$i_1 = i_2 + i_b \tag{3}$$

$$i_b + 99i_b = i_3 \text{ or } i_3 = 100i_b \tag{4}$$

Ohm's law adds three more equations for the resistors:

$$v_1 = i_1 R_1 \tag{5}$$

$$v_2 = i_2 R_2 \tag{6}$$

$$v_3 = i_3 R_3 \tag{7}$$

There are now seven equations in seven unknowns. Any method of solution might be used, but solving them by hand is instructive. The idea is to solve simpler equations for a variable and to substitute for that variable in the other equations. Here, all voltages may be replaced by current times resistance (i.e., using the Ohm's law equations). With these substitutions, the following equations replace (1) and (2):

$$3.3\text{V} - i_1 R_1 - i_2 R_2 = 0\text{V} \tag{8}$$

$$i_2 R_2 - 0.66\text{V} - i_3 R_3 = 0\text{V} \tag{9}$$

There are now four equations in four unknowns, namely (3), (4), (8), and (9). Using (3), we replace i_1 by $i_2 + i_\text{b}$ in (8):

$$3.3\text{V} - \left(i_2 + i_b\right) R_1 - i_2 R_2 = 0\text{V} \tag{10}$$

Using (4), we replace i_3 with $100 i_\text{b}$ in (9):

$$i_2 R_2 - 0.66\text{V} - 100 i_\text{b} R_3 = 0\text{V} \tag{11}$$

Equations (10) and (11) are two equations in two unknowns. The last step is to solve the simpler of these two equations for i_2 and substitute into the other equation. (Eliminating i_2 is preferred over eliminating i_b since the question asks for the value of i_2.) Here, (11) appears to be the simpler of the two equations. Solving (11) for i_b yields the following equation:

$$i_2 = \frac{0.66\text{V} + 100 i_\text{b} R_3}{R_2} \tag{12}$$

Substituting into (10) (after collecting terms multiplying i_2) results in single equation for i_b, which is found as follows:

$$3.3\text{V} - i_2 (R_1 + R_2) - i_b R_1 = 0\text{V} \tag{13}$$

or

$$3.3\text{V} - \left(\frac{0.66\text{V} + 100 i_\text{b} R_3}{R_2} \right) (R_1 + R_2) - i_b R_1 = 0\text{V} \tag{14}$$

or

$$3.3\text{V} - \frac{0.66\text{V}}{R_2} (R_1 + R_2) - \left(\frac{100 i_\text{b} R_3}{R_2} \right) (R_1 + R_2) - i_b R_1 = 0\text{V} \tag{15}$$

or

$$3.3\text{V} - \frac{0.66\text{V}}{R_2} (R_1 + R_2) - i_\text{b} \left[\left(\frac{100 R_3}{R_2} \right) (R_1 + R_2) + R_1 \right] = 0\text{V} \tag{16}$$

or

$$-i_b\left[\left(\frac{100R_3}{R_2}\right)(R_1+R_2)+R_1\right]=-3.3\text{V}+\frac{0.66\text{V}}{R_2}(R_1+R_2) \tag{17}$$

or

$$i_b\left[\left(\frac{100R_3}{R_2}\right)(R_1+R_2)+R_1\right]=3.3\text{V}-\frac{0.66\text{V}}{R_2}(R_1+R_2) \tag{18}$$

or

$$i_b\left[\left(\frac{100(30\text{k}\Omega)}{2\text{M}\Omega}\right)(1\text{M}\Omega+2\text{M}\Omega)+1\text{M}\Omega\right]=3.3\text{V}-\frac{0.66\text{V}}{2\text{M}\Omega}(1\text{M}\Omega+2\text{M}\Omega) \tag{19}$$

or

$$i_b\left[(1.5)(3\text{M}\Omega)+1\text{M}\Omega\right]=3.3\text{V}-\frac{0.66\text{V}}{2\text{M}\Omega}(3\text{M}\Omega) \tag{20}$$

or

$$i_b\left[5.5\text{M}\Omega\right]=3.3\text{V}-0.99\text{V}=2.31\text{V} \tag{21}$$

or

$$i_b=\frac{2.31\text{V}}{5.5\text{M}\Omega}=0.42\mu\text{A}\ . \tag{22}$$

At last! The remaining calculations are now straightforward. Voltage v_3 follows from (4) and (7).

$$v_3=i_3R_3=100i_bR_3=100(0.42\mu\text{A})30\text{k}\Omega=1.26\text{V} \tag{23}$$

For the power, we must determine the voltage across the dependent source. A clockwise v-loop around the outside of the circuit (or, alternatively, the upper right v-loop) will suffice.

$$3.3\text{V}-v_{\text{dep src}}-v_3=0\text{V} \tag{24}$$

or

$$v_{\text{dep src}}=3.3\text{V}-v_3=3.3\text{V}-1.26\text{V}=2.04\text{V} \tag{25}$$

Using $p = iv$ for the two sources in the box (and following the passive sign convention), we find the total power dissipation for the box.

$$p = i_b(0.66\text{V}) + 99 i_b(2.04\text{V}) = i_b[0.66\text{V} + 99(2.04\text{V})] \tag{26}$$

or

$$p = i_b[(0.66\text{V}) + 99(2.04\text{V})] = i_b[202.62\text{V}] \tag{27}$$

or

$$p = [0.42\mu\text{A}][202.62\text{V}] \approx 85.1\mu\text{W} \tag{28}$$

NOTE: A clever method for simplifying calculations in a circuit of this kind (which is modeling a bipolar transistor in the box) is to use "impedance [or resistance] multiplication". The circuit below gives the same values of i_b and v_3 as the original circuit. The idea of this circuit is that $100i_b$ flows in the original R_3, but the same voltage drop on R_3 results from using only i_b in R_3 and multiplying R_3 by 100.