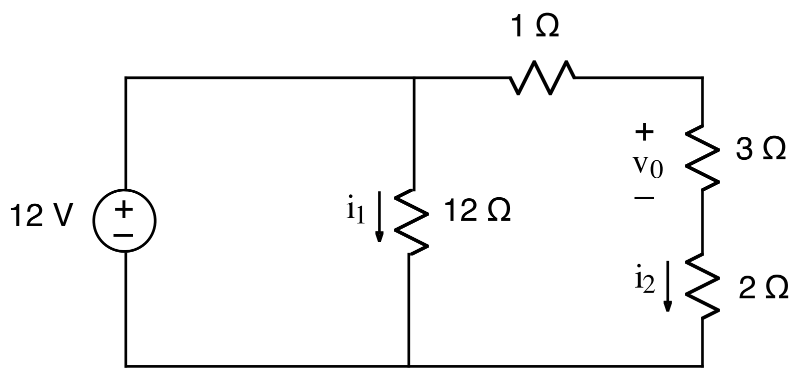
HOMEWORK #3 prob 3 solution

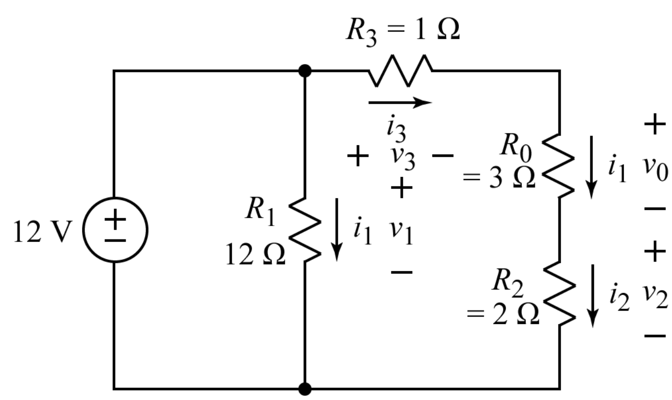
Ex:



a) Calculate i_1, i_2, and v_0.

b) Find the power dissipated for every component, including the voltage source.

SOL'N: a) We first label voltage and current for each resistor.



Starting with the voltage loops, we have the following equations:

v-loop on left: $12\,\text{V} - v_1 = 0\,\text{V}$ or $v_1 = 12\,\text{V}$

This equation says that a resistor across a v-source has that source voltage across it.

v-loop on right: $v_1 - v_3 - v_o - v_2 = 0\,\text{V}$

This loop is in the clockwise direction.

Since we have equations for the two inner loops, the outside v-loop would be redundant.

Now we consider i-sums at nodes.

At the top center node, we discover that we lack a current for the 12 V source. If we define a current for the voltage source, we add another unknown and another equation. Consequently, this gets us no closer to solving for the currents and voltages. Thus, we avoid writing a current-sum equation for the top center node.

The same argument applies to the bottom center node. Thus, this problem requires no curent-sum equations.

The next step is to equate currents in series components. Here, the same current must flow in the 1 Ω, 3 Ω, and 2 Ω resistors:

$$i_3 = i_o = i_2$$

From this point forward, we i_2 in place of i_3 and i_o. Note: if we look for such series currents at the outset, then we may eliminate some currents immediately.

Last, we use Ohm's law.

$$v_1 = i_1 \cdot 12\,\Omega \;\text{ or }\; i_1 = \frac{12\,\text{V}}{12\,\Omega} = 1\,\text{A}$$

$$v_o = i_2 \cdot 3\,\Omega$$

$$v_2 = i_2 \cdot 2\,\Omega$$

$$v_3 = i_2 \cdot 1\,\Omega$$

Note that we can solve for v_1 and i_1 separately. This will happen whenever we have different parts of the circuit that are connected in parallel directly across a *v*-source.

For the right side of the circuit, we can substitute the Ohm's law expressions into the voltage equation and solve for i_2:

$$v_1 - v_3 - v_o - v_2 = 0\,\text{V}$$

or

$$12\,\text{V} - i_2 \cdot 1\Omega - i_2 \cdot 3\Omega - i_2 \cdot 2\Omega = 0\,\text{V}$$

or

$$i_2(1\,\Omega + 3\,\Omega + 2\,\Omega) = 12\text{ V}$$

or

$$i_2 = \frac{12\text{ V}}{1\,\Omega + 3\,\Omega + 2\,\Omega} = \frac{12\text{ V}}{6\,\Omega} = 2\text{ A}$$

For v_o, we use Ohm's law:

$$v_o = i_2 \cdot 3\,\Omega = 2\text{ A} \cdot 3\,\Omega = 6\text{ V}$$

b) Power = $i \cdot v$

For resistors, $p = iv = i^2 R = \dfrac{v^2}{R}$

$$p_{12\Omega} = i_1^2 \cdot 12\,\Omega = (1\text{ A})^2 \cdot 12\,\Omega = 12\text{ W}$$

$$p_{1\Omega} = i_2^2 \cdot 1\,\Omega = (2\text{ A})^2 \cdot 1\,\Omega = 4\text{ W}$$

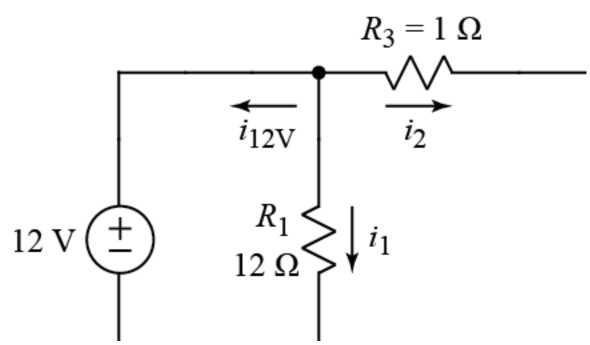
$$p_{3\Omega} = i_2^2 \cdot 3\,\Omega = (2\text{ A})^2 \cdot 3\,\Omega = 12\text{ W}$$

$$p_{2\Omega} = i_2^2 \cdot 2\,\Omega = (2\text{ A})^2 \cdot 2\,\Omega = 8\text{ W}$$

Our total power for the resistors is 36 W.

For the 12 V source, we need the current. Now that we have solved the circuit, we can use Kirchhoff's laws to find the current. Using a current source for the top center node, we have the following equation:

$$i_{12V} + i_1 + i_2 = 0\text{ A}$$



$$i_{12V} = -(i_1 + i_2) = -(1\text{ A} + 2\text{ A}) = -3\text{ A}$$

So our power for the supply is

$$p_{12\text{V}} = -3\,\text{A} \cdot 12\,\text{V} = -36\,\text{W}.$$

Total power for the circuit is –36 W + 36 W = 0 W. Note: a negative power means a source is supplying power.