EX: When a cell phone transmits, it uses small snippets of sinusoids to signal ones and zeros. Both the voltage and current will be sinusoids, and the power is the product of the voltage and current.

$$v(t) = v_{\mathrm{m}} \cos(2\pi ft)$$
$$i(t) = i_{\mathrm{m}} \cos(2\pi ft)$$
$$p(t) = v(t)i(t)$$

a) Using trigonometric identities, write $p(t)$ as a sum of a constant and a sinusoid.

b) While transmitting, the power consumed by a cellphone can be substantial. Find the average transmitting power in the transmitter if the current magnitude is $i_{\mathrm{m}} = 3$ A and the voltage is $v_{\mathrm{m}} = 2.8$ V. Hint: what term is the average value of your $p(t)$ from part (a)?

SOL'N: a) Power is the product of voltage and current.

$$p(t) = i(t) \cdot v(t) = i_{\mathrm{m}} v_{\mathrm{m}} \cos(2\pi ft)\cos(2\pi ft)$$

or, using the identity for $\cos^2(\ )$

$$p(t) = i_{\mathrm{m}} v_{\mathrm{m}} \cos^2(2\pi ft) = i_{\mathrm{m}} v_{\mathrm{m}} \left[\frac{1}{2} + \frac{1}{2}\cos(2\pi(2f)t) \right]$$

or

$$p(t) = \frac{i_{\mathrm{m}} v_{\mathrm{m}}}{2} + \frac{i_{\mathrm{m}} v_{\mathrm{m}}}{2}\cos(2\pi(2f)t).$$

Our power waveform is a cosine at twice the frequency of i and v that is shifted upwards by a constant amount.

b) Energy is the integral of power with respect to time, and average power is the energy divided by the time over which we integrate. Since the power is constantly going up and down, however, we have a dilemma that the power average actually changes with time. Nevertheless, because the positive and negative variations of power average out, the average power will approach a constant value if we use a long time interval. Indeed, we can use a single cycle of the power waveform, since the positive and negative parts of the curve will cancel out in one complete cycle.

We can compute the integral over one complete cycle of the cosine in the power expression obtained in part (a), but the cosine has equal parts positive and negative and gives a net zero contribution to our integral. So what we get is the constant term integrated over one cycle of the cosine. The following equations show the calculation (which we can largely avoid if we realize the cosine makes no contribution to the answer):

$$w(\text{one cycle}) = \int_0^{1/2f} p(t)\,dt = \int_0^{1/2f} i_{\mathrm{m}} v_{\mathrm{m}} \left[\frac{1}{2} + \frac{1}{2}\cos(2\pi(2f)t)\right] dt$$

or

$$w(\text{one cycle}) = \int_0^{1/2f} i_{\mathrm{m}} v_{\mathrm{m}} \frac{1}{2}\,dt + \int_0^{1/2f} i_{\mathrm{m}} v_{\mathrm{m}} \left[\frac{1}{2}\cos(2\pi(2f)t)\right] dt$$

or

$$w(\text{one cycle}) = i_{\mathrm{m}} v_{\mathrm{m}} \frac{1}{2} t \Big|_{t=0}^{t=1/2f} + i_{\mathrm{m}} v_{\mathrm{m}} \left[\frac{1}{2} \frac{\sin(2\pi(2f)t)}{2\pi(2f)}\right]\Bigg|_{t=0}^{t=1/2f}$$

or

$$w(\text{one cycle}) = i_{\mathrm{m}} v_{\mathrm{m}} \frac{1}{2} \frac{1}{2f} + i_{\mathrm{m}} v_{\mathrm{m}} \left[\frac{1}{2} \frac{\sin(2\pi(2f)(1/2f))}{2\pi(2f)}\right]$$

or

$$w(\text{one cycle}) = i_{\mathrm{m}} v_{\mathrm{m}} \frac{1}{4f} + i_{\mathrm{m}} v_{\mathrm{m}} \left[\frac{1}{2} \frac{\sin(2\pi)}{2\pi(2f)}\right] = i_{\mathrm{m}} v_{\mathrm{m}} \frac{1}{4f}$$

Now, to find the average, we divide by the duration of one cycle, which is $T = 1/(2f)$:

$$p_{\mathrm{ave}} = \frac{1}{\frac{1}{2f}} i_{\mathrm{m}} v_{\mathrm{m}} \frac{1}{4f} = i_{\mathrm{m}} v_{\mathrm{m}} \frac{1}{2}$$

We are left with the constant term from our answer to (a)! This was the average all along. A way to see that this is the average is to imagine smoothing out the plot of $p(t)$ as though it were made of butter. The average height of the function will be the constant term, since the cosine smooths out to an average of zero.

Plugging in the numbers given the problem gives the average power:

$$p_{\mathrm{ave}} = (3\ \mathrm{A})(2.8\ \mathrm{V})(1/2) = 4.2\ \mathrm{W}$$