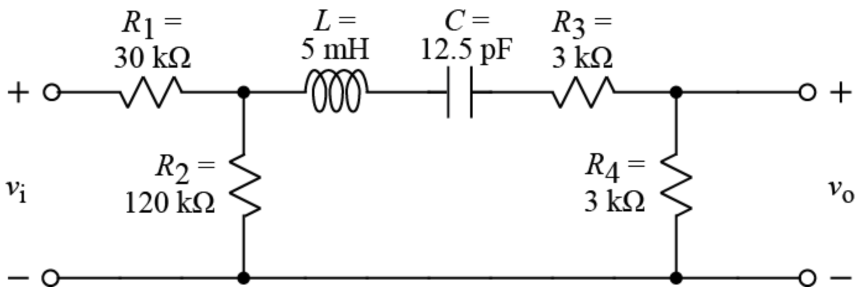
Ex:



$i_g(t) = 20\cos(2kt)$ A

a) The above is what type of filter? (circle one of the following)

band-pass band-reject

b) Find the center frequency, ω_o, of the above filter.

c) Find the maximum value of the gain, $|H(j\omega)|$, of the above filter.

d) Find the cutoff frequencies, ω_{C1} and ω_{C2}, of the above filter.

SOL'N: a) At resonance, see ω_o below, the L and C act like a wire. This reduces the impedance connecting the input signal to the output. Thus, the gain will be higher at resonance, and this is a **band-pass** filter.

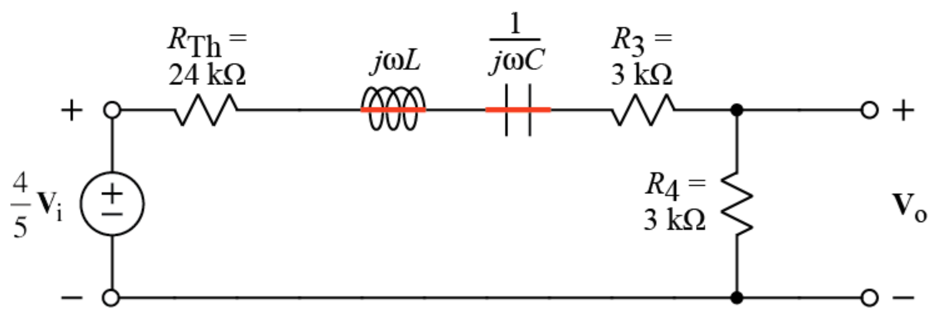
b) The center frequency occurs when the impedances of the L and C cancel out. This occurs at the resonant frequency.

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5\text{m}\cdot 12.5\text{p}}}\text{r/s} = \frac{1}{\sqrt{62.5\text{mp}}}\text{r/s}$$

or

$$\omega_o = \frac{1}{\sqrt{62500\mu\text{p}}}\text{r/s} = \frac{1}{250\text{m}\mu}\text{r/s} = 4\,\text{Mr/s}$$

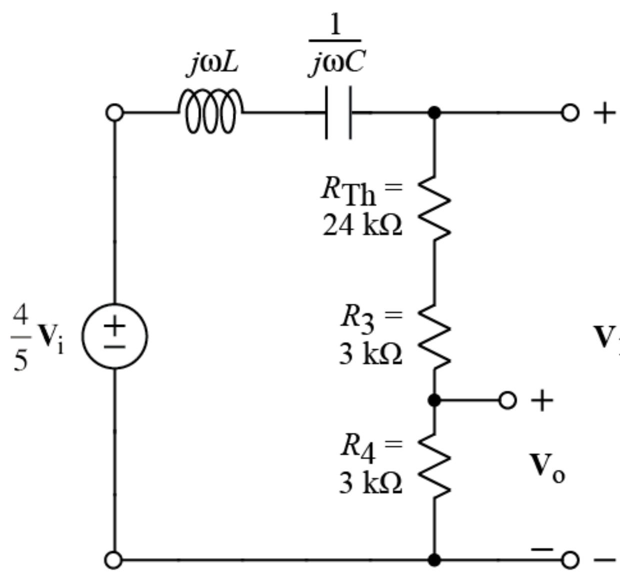
c) The maximum gain occurs at the center frequency, when the L and C act like a wire. We may also convert v_i, R_1, and R_2 to a Thevenin equivalent to simplify the calculation of the output voltage.



The transfer function is the ratio of the impedance across which $\mathbf{V}_o$ is measured.

$$|H(j\omega)| = \left|\frac{\mathbf{V}_o}{\mathbf{V}_i}\right| = \frac{4}{5}\frac{3\text{k}\Omega}{|24\text{k}\Omega + 3\text{k}\Omega + 3\text{k}\Omega|} = \frac{4}{50} = 0.08$$

d) We put resistors together and compute the transfer function for the output taken across all three resistors.



We can express the transfer function in terms of the transfer function of $\mathbf{V}_1$ relative to $\mathbf{V}_i$.

$$H(j\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\mathbf{V}_o}{\mathbf{V}_1}\frac{\mathbf{V}_1}{\mathbf{V}_i} = \frac{R_4}{R_{Th} + R_3 + R_4}\frac{\mathbf{V}_1}{\mathbf{V}_i} = \frac{1}{10}\frac{\mathbf{V}_1}{\mathbf{V}_i} = \frac{1}{10}H_1(j\omega)$$

The transfer function $H(j\omega)$ has the same cutoff frequencies as $H_1(j\omega)$, which is a standard RLC filter.

$$\omega_{C1,C2} = \pm\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

where

$$R = R_{\mathrm{Th}} + R_3 + R_4$$

or

$$\omega_{C1,C2} = \pm\frac{30\mathrm{k}}{2(5\mathrm{m})} + \sqrt{\left(\frac{30\mathrm{k}}{2(5\mathrm{m})}\right)^2 + \frac{1}{5\mathrm{m}(12.5\mathrm{p})}}$$

or

$$\omega_{C1,C2} = \pm 3\mathrm{M} + \sqrt{(3\mathrm{M})^2 + \frac{1}{62500\mu\mathrm{p}}}$$

or

$$\omega_{C1,C2} = \pm 3\mathrm{M} + \sqrt{(3\mathrm{M})^2 + \left(\frac{1}{250\mathrm{m}\mu}\right)^2}\ \mathrm{r/s}$$

or

$$\omega_{C1,C2} = \pm 3\mathrm{M} + \sqrt{(3\mathrm{M})^2 + (4\mathrm{M})^2}\ \mathrm{r/s}$$

or

$$\omega_{C1,C2} = \pm 3 + 5\ \ \mathrm{Mr/s}$$

or

$$\omega_{C1} = 2\,\mathrm{Mr/s}\ \text{and}\ \ \omega_{C2} = 8\,\mathrm{Mr/s}$$