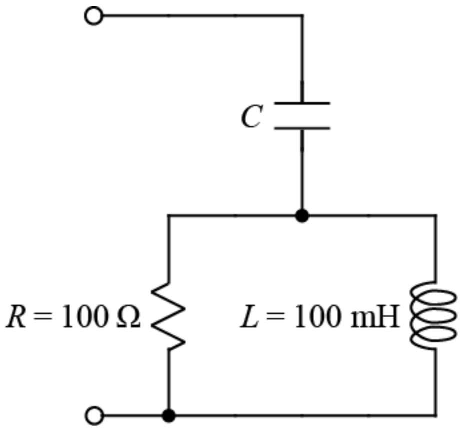
HOMEWORK #17 prob 3 solution

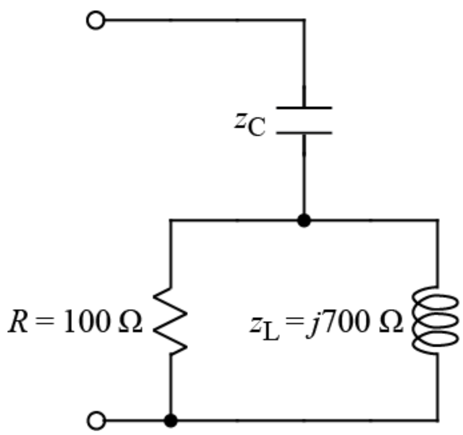
Ex:



Given ω = 7k rad/s, Find the value of C that makes the total impedance of the above circuit real. You may round off the value of C to the nearest standard value.

SOL'N: We compute the impedance of the inductor and consider the frequency domain circuit shown below.

$$z_L = j\omega L = j\,7\text{kr/s}\,(100\text{mH}) = j700\,\Omega$$



We compute the total impedance of the circuit and express it in $a + jb$ form so that we can set the imaginary part, i.e. b, to zero.

$$z_{Tot} = \frac{1}{j\omega C} + R \parallel j\omega L = -j\frac{1}{\omega C} + \frac{j\omega LR}{R + j\omega L}$$

We rationalize the parallel impedance term to we can eliminate the imaginary part in its denominator.

$$z_{Tot} = -j\frac{1}{\omega C} + \frac{j\omega LR}{R + j\omega L}\frac{R - j\omega L}{R - j\omega L} = -j\frac{1}{\omega C} + \frac{-\omega^2 L^2 R + j\omega LR^2}{R^2 + (\omega L)^2}$$

or

$$z_{\mathrm{Tot}} = -j\frac{1}{\omega C} + \frac{-\omega^2 L^2 R}{R^2 + (\omega L)^2} + j\frac{j\omega L R^2}{R^2 + (\omega L)^2}$$

We extract the imaginary part and set it to zero.

$$\mathrm{Im}[z_{\mathrm{Tot}}] = -\frac{1}{\omega C} + \frac{\omega L R^2}{R^2 + (\omega L)^2} = 0$$

Now we solve for C.

$$\frac{1}{\omega C} = \frac{\omega L R^2}{R^2 + (\omega L)^2}$$

Inverting both sides saves time.

$$\omega C = \frac{R^2 + (\omega L)^2}{\omega L R^2}$$

or

$$C = \frac{R^2 + (\omega L)^2}{\omega^2 L R^2} = \frac{100^2 + 700^2}{(7\mathrm{k})^2 (100\mathrm{m}) 100^2}\mathrm{F}$$

or, if we remove the factor of 100^2 top and bottom

$$C = \frac{1 + 49}{(7\mathrm{k})^2 (100\mathrm{m})}\mathrm{F} \approx \frac{49}{(7\mathrm{k})^2 (100\mathrm{m})}\mathrm{F} = \frac{1}{(1\mathrm{k})^2 100\mathrm{m}}\mathrm{F}$$

or

$$C \approx \frac{1}{100\mathrm{k}}\mathrm{F} = 10\,\mu\mathrm{F}$$