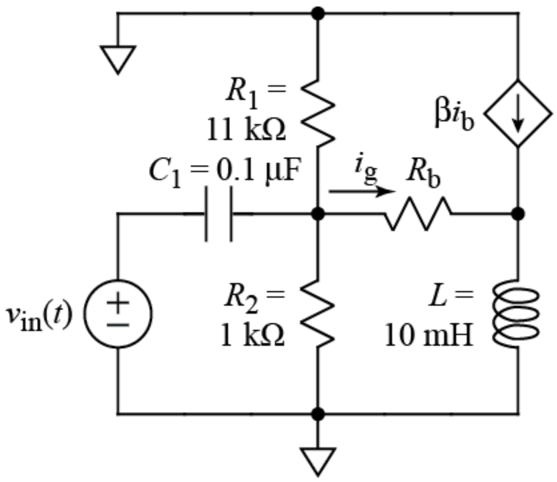
Ex: The circuit shown below is the small-signal model of an emitter follower incorporating an npn transistor (modeled by R_b and source βi_b). The input voltage in practice would be something like a music waveform. The capacitor couples the input to the input of the transistor, which is biased by R_1 and R_2 and a DC power supply that disappears in the small-signal model, (think superposition). The L represents a speaker coil (which has an impedance value that will look familiar to those who have worked with audio systems).



Note: $v_{in}(t) = 300\cos(800t)\,\text{mV}$

a) The value of R_b for the small-signal model is found by linearizing the current-versus-voltage curve for a diode in the npn transistor. The equation for the diode is as follows:

$$i_D = I_0\left(e^{v_D/v_T} - 1\right)$$

where $I_0 = 0.010$ pA is the reverse saturation current of the diode

$v_T = kT/q = 26$ mV at room temperature

v_D = voltage across diode

i_D = current in diode

The above values are deduced from a data sheet for a standard 1N914 diode (rather than an npn transistor). The URL for the diode data is http://www.mouser.com/ds/2/149/1N914-192459.pdf.

The formula for R_b is based on the slope of the nonlinear diode equation at an operating point of 0.7 V across the diode:

$$R_b = \frac{1}{\left.\frac{di_D}{dv_D}\right|_{v_D=0.70\,\mathrm{V}}}$$

Using the above formula, find the value of R_b.

b) Draw the frequency-domain circuit diagram (with numerical values for impedances and phasors [except the dependent source which is a multiple of the dependent variable]) for the circuit shown above.

SOL'N: a) The derivative is just the derivative of an exponential function, since the -1 is a constant with respect to v_D.

$$\left.\frac{di_D}{dv_D}\right|_{v_D=0.70\,\mathrm{V}} = \left.\frac{dI_0\left(e^{v_D/v_T}-1\right)}{dv_D}\right|_{v_D=0.70\,\mathrm{V}}$$

The derivative must be found before plugging in numbers. Otherwise, we would be differentiating a number and would get zero. Our derivative is like finding d/dx of Ae^{ax}, which gives aAe^{ax}.

$$\left.\frac{di_D}{dv_D}\right|_{v_D=0.70\,\mathrm{V}} = \frac{1}{v_T} I_0\, e^{v_D/v_T}\Big|_{v_D=0.70\,\mathrm{V}}$$

Now that the derivative has been found, we may plug in values:

$$\left.\frac{di_D}{dv_D}\right|_{v_D=0.70\,\mathrm{V}} = \frac{1}{26\,\mathrm{mV}}(0.010\,\mathrm{pA})e^{0.70\,\mathrm{V}/26\,\mathrm{mV}} \approx 189.5\,\mathrm{m}\Omega^{-1}$$

We now take the inverse, $(1/x)$, to get the small-signal resistance value.

$$R_b = \frac{1}{\left.\frac{di_D}{dv_D}\right|_{v_D=0.70\,\mathrm{V}}} \approx \frac{1}{189.5\,\Omega^{-1}} \approx 5.28\,\Omega$$

b) In the frequency-domain, we use phasors for voltages and currents, and impedances for resistors, capacitors, and inductors.

The phasor transform is captured by the following equation:

$$P[A\cos(\omega t + \phi)] = Ae^{j\phi} \equiv A\angle\phi$$

We apply this equation to v_s and i_s using the same units in the frequency-domain as in the time-domain. The circuit diagram, below, shows the values.

The impedances are calculated with the following formulas:

$$z_R = R \qquad z_L = j\omega L \qquad z_C = \frac{1}{j\omega C} = \frac{-j}{\omega C}$$

The value of ω = 800 r/s is found in $v_{in}(t)$. Frequency domain circuit values are shown on the circuit diagram below. We may save some effort by noting that doubling the value of L increases the impedance by a factor of two, whereas doubling the value of C decreases the impedance by a factor of two.