EX: Derive a symbolic expression for the impedance of $j\omega L + R$ in parallel with $\frac{1}{sC}$ at frequency $\omega^2 = \frac{1}{LC}$. Express the value in rectangular form, (i.e., $a + jb$ form).

SOL'N: When working with parallel impedances, it is typically easier to use the summation of conductance form of parallel impedance.

$$z = \frac{1}{\frac{1}{R + j\omega L} + \frac{1}{\frac{1}{j\omega C}}} = \frac{1}{\frac{1}{R + j\omega L} + j\omega C}$$

We clear the denominator of the denominator by multiplying top and bottom by $R + j\omega L$.

$$z = \frac{R + j\omega L}{1 + (R + j\omega L) j\omega C} = \frac{R + j\omega L}{1 - \omega^2 LC + j\omega RC}$$

Substituting the value of ω^2, the denominator simplifies.

$$z = \frac{R + j\frac{1}{\sqrt{LC}}L}{1 - \frac{1}{LC}LC + j\frac{1}{\sqrt{LC}}RC} = \frac{R + j\sqrt{\frac{L}{C}}}{jR\sqrt{\frac{C}{L}}} \cdot \frac{-j}{-j} = \frac{\sqrt{\frac{L}{C}} - jR}{R\sqrt{\frac{C}{L}}}$$

Now we divide the real and imaginary parts by the real denominator.

$$z = \frac{L}{RC} - j\sqrt{\frac{L}{C}}$$