HOMEWORK #15 prob 4 solution

Ex: Euler's formula is $e^{jx} = \cos x + j\sin x$. A cosine may be expressed in terms of complex exponentials as follows:

$$\cos x = \frac{e^{jx} + e^{-jx}}{2}$$

Use the above formula as a basis for deriving the identity for the cosine of a sum of angles.

$$\cos(x_1 + x_2) = \frac{e^{j(x_1+x_2)} + e^{-j(x_1+x_2)}}{2} = \frac{e^{jx_1}e^{jx_2} + e^{-jx_1}e^{-jx_2}}{2} = \ldots$$

SOL'N: Replace each term with its Euler's formula.

$$\cos(x_1 + x_2) = \frac{(\cos x_1 + j\sin x_1)(\cos x_2 + j\sin x_2) + (\cos x_1 - j\sin x_1)(\cos x_2 - j\sin x_2)}{2}$$

Multiply the terms.

$$\cos(x_1 + x_2) = \frac{\cos x_1 \cos x_2}{2} - \frac{\sin x_1 \sin x_2}{2} + j\frac{\cos x_1 \sin x_2}{2} + j\frac{\sin x_1 \cos x_2}{2}$$
$$\frac{\cos x_1 \cos x_2}{2} - \frac{\sin x_1 \sin x_2}{2} - j\frac{\cos x_1 \sin x_2}{2} - j\frac{\sin x_1 \cos x_2}{2}$$

The imaginary terms cancel out, and the desired trigonometric identity follows.

$$\cos(x_1 + x_2) = 2\left(\frac{\cos x_1 \cos x_2}{2} - \frac{\sin x_1 \sin x_2}{2}\right) = \cos x_1 \cos x_2 - \sin x_1 \sin x_2$$