Homework 15

1. Give numerical answers to each of the following questions:

   a) Find the value of $z = 12 - j16 + 7 + j24$.

   b) Find the magnitude of $z = 15 + j8$.

   c) Find the conjugate of $z = \frac{3 + j4}{j} \cdot \frac{-j}{3 - j4}$.

   d) Find the value of $z = \left(1 + j\sqrt{3}\right)\left(\frac{\sqrt{3}}{4} - j\frac{1}{4}\right)$.

2. Compute each of the following sums using vectors in the complex plane:

   a) $z = (1 + j3) + (-2 + j) + (1 - j3)$

   b) $z = \frac{1 + j}{2} + \frac{1 - j}{2}$

   c) $z = (5 + j12) + (-24 + j10)$

   d) $z = (1 + j0) + (-1 + j\sqrt{3}) + (-1 - j\sqrt{3}) + (1 + j0)$

3. Give numerical answers to each of the following questions:

   a) Rationalize $\frac{4375 - j15{,}000}{7 + j24}$. Express your answer in rectangular form.

   b) Find the magnitude of $\frac{1}{2} + j\frac{\sqrt{3}}{2}$.

   c) Find the real part of $\frac{(1 + j)^4}{1 + j\sqrt{3}}$.

4. Euler's formula is $e^{jx} = \cos x + j\sin x$. A cosine may be expressed in terms of complex exponentials as follows:

$$\cos x = \frac{e^{jx} + e^{-jx}}{2}$$

Use the above formula as a basis for deriving the identity for the cosine of a sum of angles.

$$\cos\left(x_1 + x_2\right) = \frac{e^{j(x_1 + x_2)} + e^{-j(x_1 + x_2)}}{2} = \frac{e^{jx_1}e^{jx_2} + e^{-jx_1}e^{-jx_2}}{2} = \ldots$$

5. If $z_1 = j$, find a complex number, z_2, such that $z_1 + z_2 = z_1 z_2$. Express z_2 in rectangular (i.e., $a + jb$) form.

Answers:

1.a) $z = 19 + j8$ b) 17 c) $\frac{7}{25} + j\frac{24}{25}$ d) $\frac{\sqrt{3}}{2} + j\frac{1}{2}$

2.a) vecs sum to j b) vecs sum to 1 on real axis c) vecs are perpendicular
d) draws an equilateral triangle

3.a) $-527 - j336$ b) 1 c) -1

4. $\cos(x_1 + x_2) = \cos(x_1)\cos(x_2) - \sin(x_1)\sin(x_2)$

5. $z_2 = \frac{1}{2} - j\frac{1}{2}$