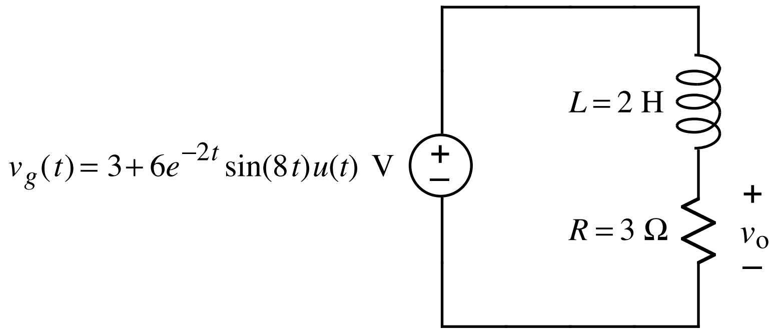
HOMEWORK #14 probs 1 and 2 solution

Ex:



Note: The 3 V in the $v_g(t)$ source is always on.

a) Write the Laplace transform, $V_g(s)$, of $v_g(t)$.

b) Draw the s-domain equivalent circuit, including source $V_g(s)$, components, initial conditions for L, and terminals for $V_o(s)$.

c) Write an expression for $V_o(s)$.

d) Apply the initial value theorem to find $\lim_{t \to 0+} v_o(t)$.

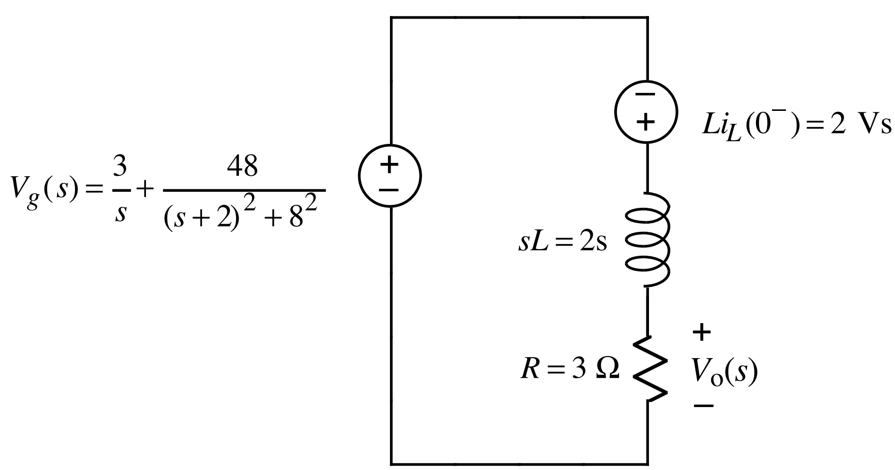
SOL'N: a) We treat the 3V as $3u(t)$:

$$V_g(s) = \frac{3}{s} + 6 \cdot \frac{8}{(s+2)^2 + 8^2} = \frac{3}{s} + \frac{48}{(s+2)^2 + 8^2}$$

b) We find initial conditions by considering the circuit at $t = 0^-$. The source at that time is a constant 3V, and the L acts like a wire.

$$i_L(t = 0^-) = \frac{3V}{3\Omega} = 1A$$

For convenience, we use a series voltage source of value $Li_L(0^-)$ for the initial conditions on the L:



c) We have a voltage-divider driven by the sum of the voltage sources:

$$V_o(s) = \left[V_g(s) + Li_L(0^-)\right]\frac{R}{sL+R} = \left[\frac{3}{s} + \frac{48}{(s+2)^2+8^2} + 2\right]\frac{3}{2s+3}$$

d) We use the initial value theorem:

$$\lim_{t\to 0+} v_o(t) = \lim_{s\to\infty} sV_o(s) = \lim_{s\to\infty} s\left[\frac{3}{s} + \frac{48}{(s+2)^2+8^2} + 2\right]\frac{3}{2s+3}$$

We multiply through by s:

$$\lim_{s\to\infty} sV_o(s) = \lim_{s\to\infty}\left[3 + \frac{48s}{(s+2)^2+8^2} + 2s\right]\frac{3}{2s+3}$$

We need a polynomial over a polynomial so we can identify the highest power of s on the top and bottom:

$$\lim_{s\to\infty} sV_o(s) = \lim_{s\to\infty}\left[\frac{[3+2s][(s+2)^2+8^2]+48s}{(s+2)^2+8^2}\right]\frac{3}{2s+3}$$

The result is as follows:

$$\lim_{s\to\infty} sV_o(s) = \frac{2s^3}{s^2}\cdot\frac{3}{2s} = 3$$

The above is the same as what we seek:

$$\lim_{t\to 0+} v_o(t) = 3 \text{ V}$$