HOMEWORK #13 prob 2a solution

Ex: Find $f(t)$ if $F(s) = \dfrac{5s - 62}{s^2 + 6s + 58} - \dfrac{8}{s}$.

Sol'n: First, we find the roots of the quadratic denominator. Since half the coefficient, 6, of s when squared is less than the constant term, 58, the roots are complex.

$$s = -a \pm j\omega$$

To find a and ω, we expand the denominator as follows:

$$s^2 + 6s + 58 = (s + a)^2 + \omega^2 = s^2 + 2as + a^2 + \omega^2$$

From the coefficient of s, we find a:

$$a = \frac{6}{2} = 3$$

Using this value of a, we solve for ω:

$$a^2 + \omega^2 = 3^2 + \omega^2 = 58$$

or

$$\omega^2 = 49$$

or

$$\omega = 7$$

We write the first term of $F(s)$ as a sum of a decaying cosine and sine (in the time domain):

$$F(s) = \frac{K_1(s + 3) + K_2\omega}{s^2 + 6s + 58} - \frac{8}{s}$$

Equating the numerators by matching the coefficients of each power of s, starting with the highest, yields the values of K_1 and K_2:

$$K_1(s + a) + K_2\omega = K_1 s + K_1 3 + K_2 7 = 5s - 62$$

or

$$K_1 s = 5s \quad \text{and} \quad K_1 3 + K_2 7 = -62$$

or

$$K_1 = 5 \quad \text{and} \quad K_2 = -11$$

Now we can write $F(s)$ in a form that allows us to invert it directly:

$$F(s) = 5\frac{s+3}{s^2+6s+58} - 11\frac{7}{s^2+6s+58} - \frac{8}{s}$$

Taking the inverse transform yields the final answer:

$$f(t) = 5e^{-3t}\cos(7t) - 11e^{-3t}\sin(7t) - 8u(t)$$