HOMEWORK #12 prob 3b solution

EX: Find the inverse Laplace transform for the following expression:

$$F(s) = \frac{7}{(s^2 + 6s + 58)}$$

SOL'N: Because the constant term of the quadratic in the denominator is larger than the square of half the middle coefficient, the quadratic has complex roots.

$$s^2 + 6s + 58 = (s + 3 + j7)(s + 3 - j7) = (s + a + j\omega)(s + a - j\omega)$$

or

$$s^2 + 6s + 58 = (s + 3)^2 + 7^2 = (s + a)^2 + \omega^2$$

We observe that $F(s)$ has the form of a transformed decaying sine:

$$F(s) = \frac{7}{(s^2 + 6s + 58)} = \frac{\omega}{(s + a)^2 + \omega^2} = \mathcal{L}\left[e^{-at}\sin(\omega t)\right]$$

Thus, we can write down our answer directly:

$$f(t) = e^{-3t}\sin(7t)u(t)$$

NOTE: We add $u(t)$ to indicate that the value in the time domain is unknown.