

EX: Find the Laplace transforms of the following waveform:

$$\frac{e^{-2t}}{t}$$

SOL'N: We apply the identity for "divide by t ":

$$\mathcal{L}\left[\frac{v(t)}{t}\right] = \int_s^\infty V(\underline{s})d\underline{s}$$

This translates into the following equation:

$$\mathcal{L}\left[\frac{e^{-at}}{t}\right] = \int_s^\infty \mathcal{L}[e^{-at}]d\underline{s} = \int_s^\infty \frac{1}{\underline{s}+a}d\underline{s} = \ln \underline{s} \Big|_s^\infty = \infty - s = \infty$$

We observe that the Laplace transform does not exist. The explanation for this result is that the function we are transforming goes to infinity at $t = 0$, and the area under this function is infinite near $t = 0$. (The function behaves like $1/t$ near $t = 0$, and the integral evaluates to $\ln(0) = -\infty$ at $t = 0$.)