

Ex: Find the Laplace transforms of the following waveform:

$$(t^2 - 1)u(t - 1)$$

SOL'N: To create a form that allows us to apply the time delay identity, we replace t with $(t - 1 + 1)$ in the part preceding the step function:

$$([(t - 1) + 1]^2 - 1)u(t - 1)$$

Now we apply the delay identity:

$$\mathcal{L}\{f(t - a)u(t - a)\} = e^{-as} \mathcal{L}\{f(t)\}$$

This means we replace $t - 1$ with t before taking the Laplace transform:

$$\mathcal{L}\{([(t - 1) + 1]^2 - 1)u(t - a)\} = e^{-s} \mathcal{L}\{[t + 1]^2 - 1\}$$

Expanding the quadratic term yields the following result:

$$e^{-s} \mathcal{L}\{[t + 1]^2 - 1\} = e^{-s} \mathcal{L}\{t^2 + 2t\} = e^{-s} \left(\frac{2}{s^3} + \frac{2}{s^2} \right) = 2e^{-s} \left(\frac{s + 1}{s^3} \right)$$