Homework 11

1. Sketch each of the following functions and then express each of them as a summation of products of common functions and step functions.

a) $$f(t) = \begin{cases} 0 & t \le 1 \\ 2(t-1)^2 & 1 \le t \le 2 \\ 2 & 2 \le t \le 5 \\ 2e^{5-t} & 5 \le t \le 8 \\ 0 & 8 < t \end{cases}$$

b) $$f(t) = \begin{cases} 0 & t < 1 \\ 4e^{-t/3}\cos(\pi t) & 1 \le t \le 4 \\ 0 & 4 < t \end{cases}$$

2. Compute the Laplace transform of the following functions by calculating the integral expression for the Laplace transform (step-by-step by hand):

a) $f(t) = u(t) - u(t-1)$ where $u(t)$ is the unit step function: $u(t) = \begin{cases} 0 & t < 0 \\ 1 & 0 \le t \end{cases}$

b) $$f(t) = \begin{cases} 0 & t < 0 \\ e^{-2t} & 0 \le t \end{cases}$$

c) $$f(t) = \begin{cases} 0 & t < 4 \\ e^{-2(t-4)} & 4 \le t \end{cases}$$

3. Using algebraic simplification and a table of Laplace transform pairs, find the Laplace transform of each of the following functions:

a) $f(t) = 3e^{-(t-1)}$

b) $f(t) = \cos(2\pi t + 30°)$

c) $f(t) = 4 + t$

4. Using algebraic simplification and a table of Laplace transform pairs, find the inverse Laplace transform of each of the following functions:

a) $F(s) = 3e^{-(s-1)}$

b) $F(s) = \frac{1}{s} + \frac{3}{s^2+4}$

c) $F(s) = \frac{s+5}{s^2+2s+5}$

5. Find the values of the following expressions:

a) $\int_{-1}^{1} \delta(t)\,dt$

b) $\int_{-\infty}^{\infty} t^2 \delta(t-2)\,dt$

c) $f(t) = \begin{cases} 0 & t \le -1 \\ \int_{-1}^{t} \tau^2 \delta(\tau-2)\,d\tau & t > -1 \end{cases}$

d) $\int_{-\infty}^{\infty} \delta(t/2)\,dt$

e) $\int_{-\infty}^{t} \left(\int_{-\infty}^{T} 2\delta(\tau)\,d\tau \right) dT$

Answers:

1.a)
$$f(t)=2(t-1)^2[u(t-1)-u(t-2)] + 2[u(t-2)-u(t-5)] + 2e^{t-5}[u(t-5)-u(t-8)]$$
or
$$f(t)=2(t-1)^2u(t-1)+\left[2t(2-t)\right]u(t-2) + \left[2\{e^{t-5}-1\}\right]u(t-5)-2e^{t-5}u(t-8)$$

2.a) $\frac{1}{s}-\frac{e^{-s}}{s}$ (you must show the integration steps) c) $F(s)=\frac{e^{-4s}}{s+2}$

3.a) Hint: $3e^{-(t-1)}=3e\cdot e^{-t}$ b) Hint: use identity for $\cos(A+B)$ c) $F(s)=\frac{4}{s}+\frac{1}{s^2}$

4.a) $\mathcal{L}^{-1}\{3e^{-(s-1)}\}=3e\mathcal{L}^{-1}\{e^{-s}\}=3e\delta(t-1)$

b) Hint: $3=\frac{3}{2}\cdot 2$

c) Hint: $s+5=(s+1)+2(2)$ split the expression into two fractions

5.a) 1

b) 4

c) Hint: $\int_{-\infty}^{\infty} f(\tau)\delta(\tau-a)\,d\tau = f(a)$

d) Hint: use a change of variables to $\tau=\frac{t}{2}$

e) Hints: $\int_{-\infty}^{t}\delta(t)\,dt=u(t)$ and $\int_{-\infty}^{t} f(t)\,dt =$ Area under $f(t)$ to the left of t