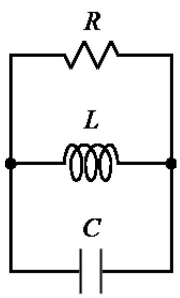
Ex:



R = 0.5 Ω

L = 1.5 μH

C = 1.5 μF

a) Find the characteristic roots, s_1 and s_2, for the above circuit.

b) Is the circuit over-damped, critically-damped, or under-damped? Explain.

c) If the L and C values in the circuit are decreased by a factor of two, (and R remains the same), what kind of damping results?

SOL'N: a) We have a parallel RLC circuit. We use the equations for roots of a parallel RLC.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$$

where

$$\alpha = \frac{1}{2RC} = \frac{1}{2(0.5)1.5\mu s} = \frac{1M}{1.5s} = \frac{2}{3}\text{Mr/s} \approx 667\,\text{kr/s}$$

and

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(1.5\mu)(1.5\mu)s^2} = \left(\frac{2}{3}\text{Mr/s}\right)^2$$

Using component values, we compute the numerical values of the roots.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -\frac{2}{3}\text{Mr/s} \pm \sqrt{\left(\frac{2}{3}\text{Mr/s}\right)^2 - \left(\frac{2}{3}\text{Mr/s}\right)^2}$$

or

$$s_{1,2} = -\frac{2}{3}\text{Mr/s}$$

b) The roots are both the same, so we have critical damping.

c) We reduce L and C by a factor of two and see what happens.

$$\alpha_{\text{new}} = \frac{1}{2RC/2} = 2\cdot\frac{2}{3}\,\text{Mr/s} \approx 1.33\,\text{Mr/s}$$

$$\omega^2_{\text{o\_new}} = \frac{1}{(L/2)(C/2)} = \frac{4}{(1.5\mu)(1.5\mu)\text{s}^2} = \left(2\cdot\frac{2}{3}\,\text{Mr/s}\right)^2$$

We see that the square root will once again be zero.

$$\sqrt{\alpha^2_{\text{new}} - \omega^2_{\text{o\_new}}} = \sqrt{\left(2\cdot\frac{2}{3}\,\text{Mr/s}\right)^2 - \left(2\cdot\frac{2}{3}\,\text{Mr/s}\right)^2} = 0\,\text{Mr/s}$$

Thus, the two roots are again the same, and we again have critical damping.

$$s_{1,2} = -\frac{4}{3}\,\text{Mr/s}$$