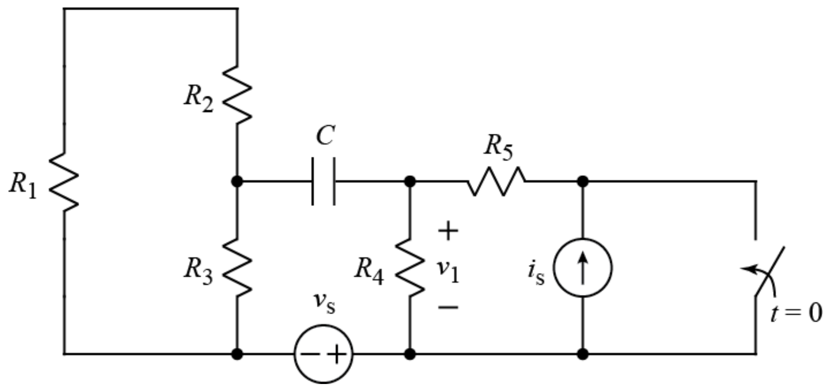
Ex:



After being open for a long time, the switch closes at $t = 0$.

$v_s = 28$ V $\quad i_s = 112$ mA $\quad C = 2$ nF

$R_1 = 43\,\Omega \quad R_2 = 47\,\Omega \quad R_3 = 120\,\Omega \quad R_4 = 750\,\Omega \quad R_5 = 1\,\text{k}\Omega$

a) Calculate the energy stored in the capacitor at $t = 0^+$.

b) Write a numerical time-domain expression for $v_1(t > 0)$, the voltage across R_4.

SOL'N: a) To find the charge on the capacitor at $t = 0^+$, we consider time $t = 0^-$. At $t = 0^-$, we treat the C as an open circuit, and the switch is open. In this case, R_5 is in series with current source i_s and may be ignored. Since C is open, current i_s flows in R_4, giving rise to a voltage across R_4. The three resistors on the left collapse into one resistor, and we observe that no current flows in v_s. We conclude that the voltage across C is the voltage drop across R_4 and v_s.

$$v_C(0^-) = v_s + i_s R_4 = 28\,\text{V} + 112\,\text{mA} \cdot 750\,\Omega = 28\,\text{V} + 84\,\text{V} = 112\,\text{V}$$

The voltage on the capacitor right after the switch moves is the same as the voltage just before the switch moves:

$$v_C(0^+) = v_C(0^-) = 112\,\text{V}$$

Now we can calculate the desired energy:

$$w_C(0^+) = \frac{1}{2} C v_C^2 = \frac{1}{2} 2\text{n}(112)^2\,\text{J} = 12.544\,\mu\text{J}$$

b) We use the general formula for RC solutions:

$$v_1(t>0) = v_1(t\to\infty) + [v_1(0^+) - v_1(t\to\infty)]e^{-\frac{t}{R_{Th}C}}$$

First we consider time approaching infinity. The capacitor will act like an open circuit, and the switch will be closed, shorting out i_s. All of i_s will flow in the switch, rather than through R_4 and R_5. No current in R_4 means we have no voltage drop across R_4.

$$v_1(t\to\infty) = 0\,\text{V}$$

Second, we consider $t = 0^+$. We replace the three resistors on the left side with one resistor:

$$R_{Eq} = (R_1 + R_2) \,\|\, R_3 = (43\,\Omega + 47\,\Omega) \,\|\, 120\,\Omega = 90\,\Omega \,\|\, 120\,\Omega$$

or

$$R_{Eq} = 30\,\Omega \cdot 3 \,\|\, 4 = 30\,\Omega \cdot \frac{12}{7}$$

We may ignore i_s since it is shorted out by the switch, and we model the C as a voltage source of value $v_C(0^+) = v_C(0^-) = 112\,\text{V}$. We are left with resistors and two voltage sources. Our value of v_1 is given by a voltage divider:

$$v_1(0^+) = (v_C - v_s)\frac{R_4 \,\|\, R_5}{R_{Eq} + R_4 \,\|\, R_5} = 84\,\text{V}\frac{250\,\Omega \cdot \frac{12}{7}}{280\,\Omega \cdot \frac{12}{7}} = 75\,\text{V}$$

Third, we find R_{Th} by turning off the independent sources and looking into the circuit from the terminals where C is attached.

$$R_{Th} = R_{Eq} + R_4 \,\|\, R_5 = 30\,\Omega \cdot \frac{12}{7} + 250\,\Omega \cdot 3 \,\|\, 4 = 30\,\Omega \cdot \frac{12}{7} + 250\,\Omega \cdot \frac{12}{7}$$

or

$$R_{Th} = 280\,\Omega \cdot \frac{12}{7} = 40\,\Omega \cdot 12 = 480\,\Omega$$

The time constant is $\tau = R_{Th}C$.

$$\tau = 480\,\Omega \cdot 2\,\text{nF} = 960\,\text{ns}$$

Fourth and last, we plug values into the general solution:

$$v_1(t>0) = 0 + [75 - 0]e^{-\frac{t}{960\,\text{ns}}}\ \text{V} = 75e^{-\frac{t}{960\,\text{ns}}}\ \text{V}$$