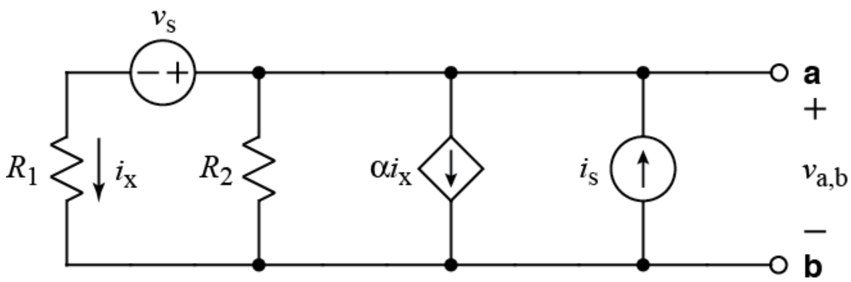
Homework 10

1.



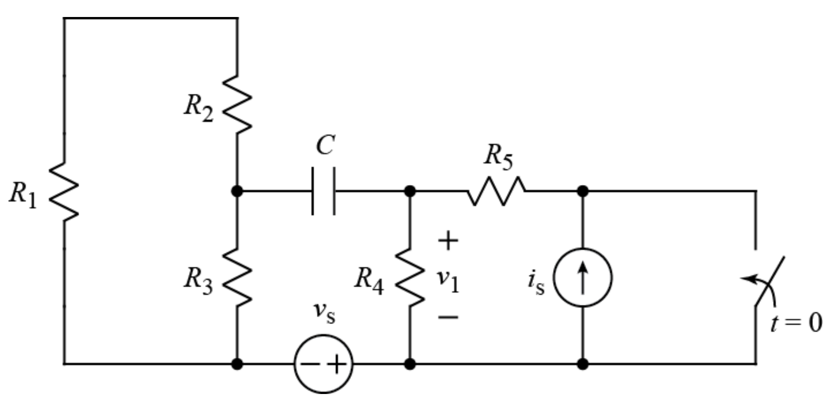
a) Using superposition, derive an expression for $v_{a,b}$ that contains no circuit quantities other than i_s, v_s, R_1, R_2, and α. Current i_x must not appear in your solution. **Note:** $\alpha \geq 0$.

b) Make a consistency check on your expression for $v_{a,b}$ by setting resistors and sources to numerical values for which the value of $v_{a,b}$ is obvious. State the values of resistors and sources for your consistency check, and show that your expression for $v_{a,b}$ is satisfied for these values. (In other words, plug the values into your expression from part (a) and show that it agrees with the value from your consistency check.)

c) Find the Thevenin equivalent circuit at terminals **a** and **b**. Express the Thevenin voltage, v_{Th}, and Thevenin resistance, R_{Th} in terms of no circuit quantities other than i_s, v_s, R_1, R_2, and α. i_x must not appear in your solution. **Note:** $\alpha \geq 0$.

d) Find an expression for the value of R_L connected from **a** to **b** that would absorb maximum power. Your answer must be written in terms of no circuit quantities other than i_s, v_s, R_1, R_2, and α. **Note:** $\alpha \geq 0$.

2.



After being open for a long time, the switch closes at $t = 0$.

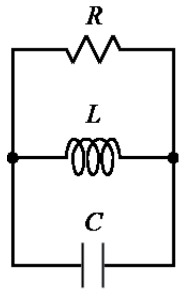
$v_s = 28$ V $\quad i_s = 112$ mA $\quad C = 2$ nF

$R_1 = 43\ \Omega \quad R_2 = 47\ \Omega \quad R_3 = 120\ \Omega \quad R_4 = 750\ \Omega \quad R_5 = 1\ \text{k}\Omega$

a) Calculate the energy stored in the capacitor at $t = 0^+$.

b) Write a numerical time-domain expression for $v_1(t > 0)$, the voltage across R_4.

3.



R = 0.5 Ω

$L = 1.5\ \mu H$

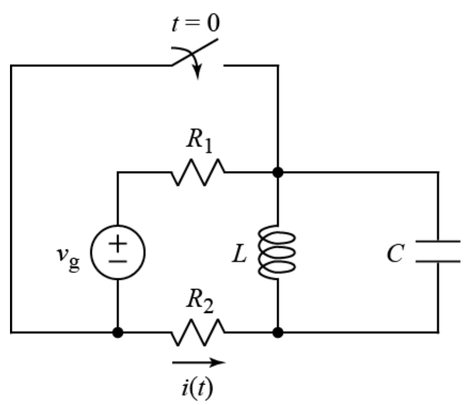
$C = 1.5\ \mu F$

a) Find the characteristic roots, s_1 and s_2, for the above circuit.

b) Is the circuit over-damped, critically-damped, or under-damped? Explain.

c) If the L and C values in the circuit are decreased by a factor of two, (and R remains the same), what kind of damping results?

4.



After being open for a long time, the switch closes at $t = 0$.

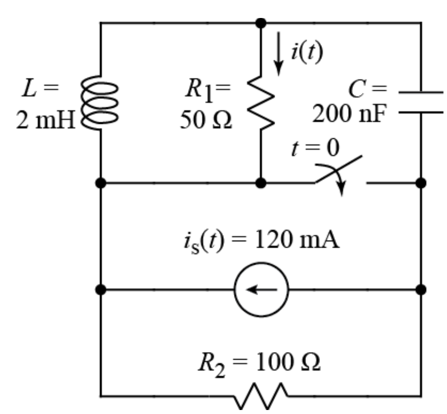
a) Give expressions for the following in terms of no more than v_g, R_1, R_2 L, and C:

$$i(t=0^+) \qquad \text{and} \qquad \left.\frac{di(t)}{dt}\right|_{t=0^+}$$

b) Find the numerical value of R_2 given the following information:

$$R_1 = 150\ \Omega \qquad L = 40\ \text{mH} \qquad C = 3.2\ \mu\text{F} \qquad \alpha = 1250\ \text{r/s} \qquad \omega_d = 2500\ \text{r/s}$$

5.



After being open for a long time, the switch closes at $t = 0$.

a) Find characteristic roots and whether $i(t)$ is under-, over-, or critically-damped.

b) Write a numerical time-domain expression for $i(t)$, $t > 0$, the current through R_1.

Answers:

1.a) $v_{a,b} = v_s \dfrac{R_2(1+\alpha)}{R_1 + R_2(1+\alpha)} + i_s \dfrac{R_1 R_2}{(1+\alpha)R_2 + R_1}$ c) $R_{Th} = \dfrac{R_1 R_2}{(1+\alpha)R_2 + R_1}$ d) $R_L = R_{Th}$

2.a) $w_C(0^+) = 12.544\,\mu\text{J}$ b) $v_1(t>0) = 75e^{-\frac{t}{960\,\text{ns}}}\ \text{V}$

3.a) Duplicate roots = -2/3 Mr/s b) critically damped c) critically damped

4.a) Partial answer: $\left.\dfrac{di(t)}{dt}\right|_{t=0^+} = \dfrac{1}{R_2}\dfrac{i_C(t=0^+)}{C} = -\dfrac{v_g}{R_2 C(R_1+R_2)}$. b) R_2 = 125 Ω.

5.a) $s_{1,2} = -50\,\text{kr/s}$, critically damped b) $i(t) = 240\,\text{mA}\,e^{-50\text{k}\,t} - 12\,\text{kA/s}\,t\,e^{-50\text{k}\,t}$