Ex:

a) Find $v(t)$ for $t \geq 0$ if $V(s) = \frac{1}{3} \cdot \frac{s+13.5}{s^2+18s+90}$.

b) Find $\lim_{t \to 0^+} v(t)$ if $V(s) = \frac{5(s+0.8)(s+0.6)+4}{(s+2)\left[(s+0.6)^2+(0.8)^2\right]}$.

SOL'N: a) We first find the roots of the denominator.

$$V(s) = \frac{1}{3} \cdot \frac{s+13.5}{s^2+18s+90} = \frac{1}{3} \cdot \frac{s+13.5}{(s+9)^2+3^2}$$

We have complex roots, meaning we must have a decaying cosine and/or sine. We can do partial fractions with coefficients for both the cosine and sine terms as follows and avoid complex numbers.

$$\mathcal{L}\left\{Ae^{-9t}\cos(3t) + B\sin e^{-9t}\sin(3t)\right\} = \frac{A(s+9)}{(s+9)^2+3^2} + \frac{B(3)}{(s+9)^2+3^2}$$

We can put the partial fraction terms over a common denominator so we can match up the numerator with the given $V(s)$. Note that the factor of 1/3 is left out until the end.

$$V(s) = \frac{1}{3} \cdot \frac{s+13.5}{(s+9)^2+3^2} = \frac{1}{3} \cdot \frac{A(s+9)+B(3)}{(s+9)^2+3^2}$$

We equate the numerators and match the coefficients of each power of s.

$$s+13.5 = A(s+9)+B(3) = As+(9A+3B)$$

From the coefficient for s, we find A:

$$A = 1$$

From the constant term, we have a second equation involving B.

$$13.5 = 9A+3B$$

Substituting for A, we have an equation for B.

$$13.5 = 9+3B$$

or

$B = 1.5$

We multiply the inverse transform by $u(t)$, just to remind ourselves that we don't know the value of $v(t)$ for $t < 0$. Remember to multiply by 1/3!

$$\left[\frac{1}{3}e^{-9t}\cos(3t) + \frac{1}{2}\sin e^{-9t}\sin(3t)\right]u(t)$$

b) We use the initial value theorem:

$$\lim_{t\to 0^+} v(t) = \lim_{s\to\infty} sV(s)$$

Thus,

$$\lim_{t\to 0^+} v(t) = \lim_{s\to\infty} sV(s) = \lim_{s\to\infty} s\frac{5(s+0.8)(s+0.6)+4}{(s+2)\left[(s+0.6)^2+(0.8)^2\right]}$$

or, using the only the highest power of s (which dominates as s approaches infinity) in sums, we find the value of the limit.

$$\lim_{t\to 0^+} v(t) = \lim_{s\to\infty} sV(s) = \lim_{s\to\infty} s\frac{5s^2}{s^3} = 5$$