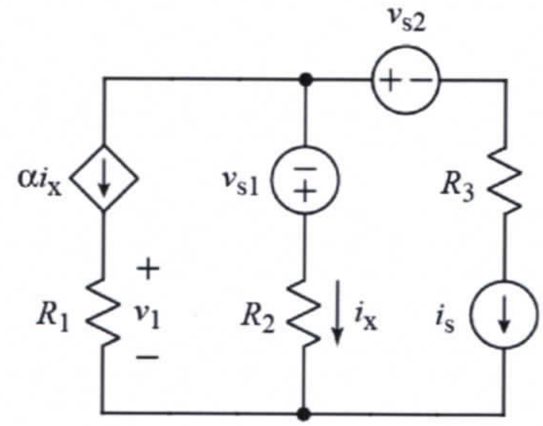
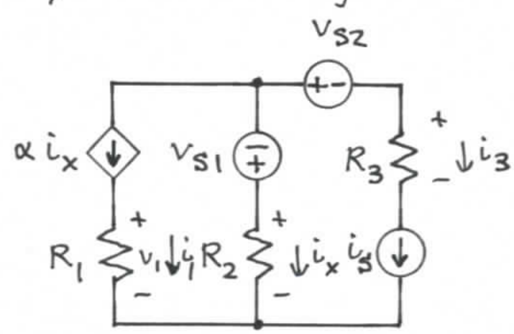
Ex:



a) Derive an expression for v_1. The expression must not contain more than the circuit parameters v_{s1}, v_{s2}, i_s, R_1, R_2, R_3 and α. **Note:** $\alpha > 0$.

b) Derive an expressions for the power dissipated by resistor R_3. The expression must not contain more than the circuit parameters v_{s1}, v_{s2}, i_s, R_1, R_2, R_3 and α.

sol'n: a) Using Kirchhoff's laws and Ohm's law, we start by labeling v and i for each R. One way of labeling the circuit is shown below.



We have no v-loops, since every v-loop would pass through a current source.

For the top node, we have this i-sum:

$$\alpha i_x + i_x + i_s = 0A$$

We can solve the i-sum eq'n for i_x.

$$(1+\alpha)\, i_x = -i_s$$

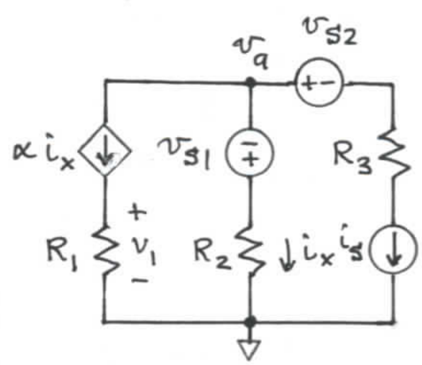
or

$$i_x = \frac{-i_s}{(1+\alpha)}$$

To find v_1, we use Ohm's law. R_1 is in series with the αi_x source. Thus, v_1 is $\alpha i_x \cdot R_1$:

$$v_1 = \alpha i_x R_1 = \frac{-\alpha i_s R_1}{1+\alpha}$$

As an alternative, we may use the Node-V method. In the circuit below, we have a reference on the bottom and node v_a on the top.



Eq'n for node v_a: $\alpha i_x + \dfrac{v_a + v_{s1}}{R_2} + i_s = 0\,A$

We replace i_x with an expression in terms of v_a:

$$i_x = \frac{v_a + v_{s1}}{R_2}$$

New node v_a eq'n: $\alpha \dfrac{v_a + v_{s1}}{R_2} + \dfrac{v_a + v_{s1}}{R_2} + i_s = 0A$

Now we do the algebra to find v_a.

$$v_a \frac{(1+\alpha)}{R_2} = -i_s - \frac{(1+\alpha)}{R_2} v_{s1}$$

or

$$v_a = -i_s \frac{R_2}{1+\alpha} - v_{s1}$$

From v_a, we find i_x, then αi_x, then v_1.

$$i_x = \frac{v_a + v_{s1}}{R_2} = -\frac{i_s}{1+\alpha}$$

then

$$\alpha i_x = -\frac{\alpha i_s}{1+\alpha}$$

then

$$v_1 = \alpha i_x R_1 = -\frac{\alpha i_s}{1+\alpha} R_1.$$

b) Power is $i^2 R$ for a resistor. The i is i_s for R_3.

$$P_{R3} = i_s^2 R_3$$