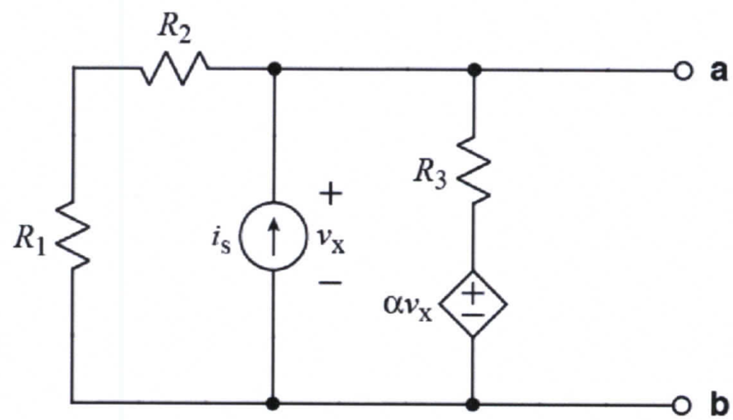
ECE 2240 **HOMEWORK #6 probs 2-4 solution**
N. Cotter

Ex:



Find the Thevenin equivalent circuit at terminals a-b.

a) Find the Thevenin equivalent circuit at terminals a-b. v_X must not appear in your solution. The expression must not contain more than circuit parameters α, R_1, R_2, R_3, and i_S. **Note:** $0 < \alpha < 1$.

b) Make at least one consistency check (other than a units check) on your expression for part (a). In other words, choose component values that make the answer obvious, and verify that your answer to part (a) gives that obvious answer. State the values of resistors and sources for your consistency check.

c) Find the Norton equivalent of the circuit

sol'n: a) It is always the case that $v_{Th} = v_{a,b}$ no load. We consider first a node-voltage solution. If we place a reference on the bottom rail, then v_{Th} is the voltage for the node consisting of the wire on the top right, which is connected to terminal a. We sum the currents flowing out of this node.

$$v_{Th}/(R_1+R_2) + -i_S + (v_{Th} - \alpha v_{Th})/R_3 = 0 \text{ A}$$

Note: $v_x = v_{Th}$ so $\alpha v_x = \alpha v_{Th}$

Now we solve for v_{Th}:

$$v_{Th}\left(\frac{1}{R_1+R_2} + \frac{1-\alpha}{R_3}\right) = i_s$$

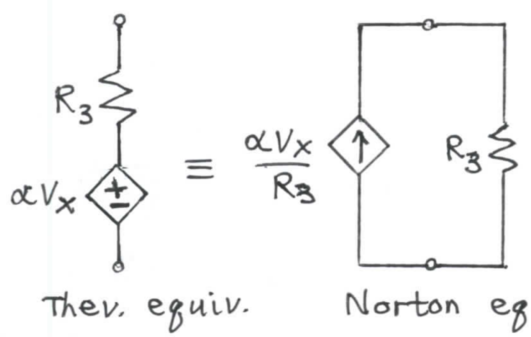
Multiplying both sides by $(R_1+R_2)R_3$ gives

$$v_{Th}\left[R_3 + (1-\alpha)(R_1+R_2)\right] = i_s(R_1+R_2)R_3$$

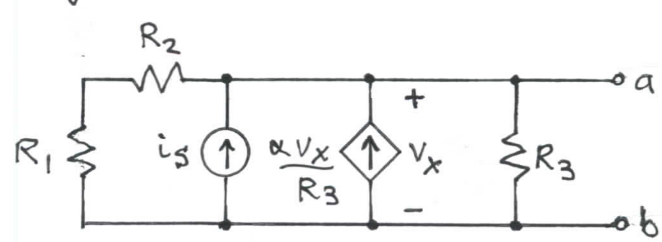
or

$$v_{Th} = \frac{i_s(R_1+R_2)R_3}{R_3+(1-\alpha)(R_1+R_2)}$$

An alternate approach (suggested by Norm Gifford) is to first replace the dependent source and R_3 with a Norton equivalent.



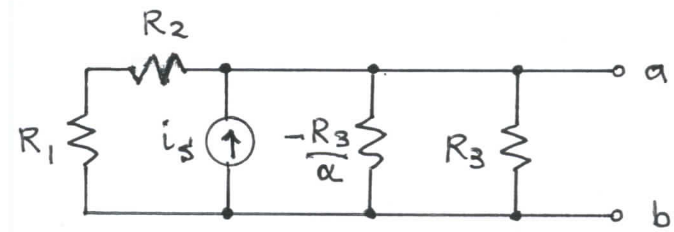
With the Norton equivalent, we have the following circuit:



We have both the voltage and current for the dependent source as functions of v_x, allowing us to find an equivalent R value.

$$R_{eq} = -\frac{V_x}{\frac{\alpha V_x}{R_3}} = -\frac{R_3}{\alpha}$$

Our new circuit model:



We now find $v_{Th} = v_{a,b}$ no load using Ohm's law:

$$v_{Th} = i_s \cdot (R_1 + R_2) \| -\frac{R_3}{\alpha} \| R_3$$

We compare this to the previous answer by simplifying the expression for parallel resistances.

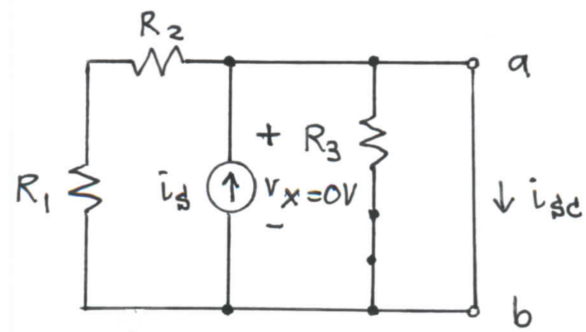
$$(R_1+R_2) \| -\frac{R_3}{\alpha} \| R_3 = \frac{1}{\frac{1}{R_1+R_2} - \frac{\alpha}{R_3} + \frac{1}{R_3}}$$

$$= \frac{(R_1+R_2) R_3}{R_3 + (1-\alpha)(R_1+R_2)}$$

So $v_{Th} = i_s \dfrac{(R_1 + R_2) R_3}{R_3 + (1-\alpha)(R_1 + R_2)}$ as before.

To find R_{Th}, we can short the output from a to b, find the short circuit current, i_{sc}, and then use $R_{Th} = v_{Th} / i_{sc}$.

When we short a to b, we have $v_x = 0V$, turning the dependent source into a wire.



Note: $v_x = 0V$ since we have a wire from a to b.

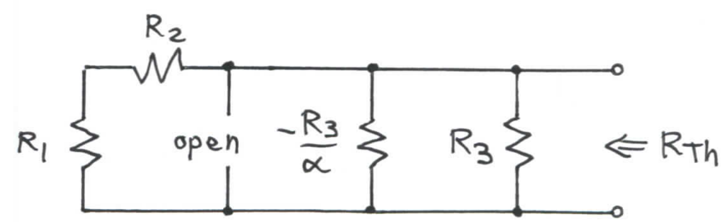
We have a current divider, but all the current will flow in the short circuit (wire) since it has zero resistance.

$$i_{sc} = i_s$$

So $$R_{Th} = \frac{v_{Th}}{i_{sc}} = \frac{v_{Th}}{i_s} = \frac{(R_1+R_2)\,R_3}{R_3+(1-\alpha)(R_1+R_2)}$$

An alternate approach is to observe that the R_{eq} that replaced the dependent source is valid regardless of what linear circuit we connect across a and b. That is, v_x is still across the Norton equivalent source $-\alpha v_x / R_3$.

In that case, we can turn off i_s and look in from the a and b terminals to find R_{Th}.



So $R_{Th} = (R_1 + R_2) \,\|\, -\frac{R_3}{\alpha} \,\|\, R_3$ as before.

b) To make a consistency check, we try to find component values that make the Thevenin equivalent obvious.

One trivial consistency check is to set $i_s = 0$ A, which causes the circuit to have voltages and currents that are all equal to zero. Thus, $v_{Th} = 0$ V. We check our answer to (a) and find that it gives $v_{Th} = 0$ V when $i_s = 0$ A. So this consistency check is satisfied. ✓

Another consistency check is $R_1 + R_2 = 0\,\Omega$. That is, $R_1 + R_2 =$ wire. In this case, we have a short circuit from a to b. Thus, $v_{Th} = 0$ V and $R_{Th} = 0\,\Omega$. When we substitute $R_1 + R_2 = 0\,\Omega$ into our answer to (a), we have $v_{Th} = \dfrac{i_s \cdot 0\Omega \cdot R_3}{R_3 + (1-\alpha)\cdot 0\Omega} = 0\,V$

and $R_{Th} = \dfrac{0\Omega \cdot R_3}{R_3 + (1-\alpha)\cdot 0\Omega} = 0\,\Omega$. ✓

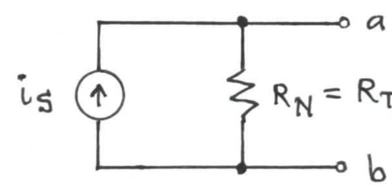
Another consistency check (suggested by a student) is to set $\alpha = 1$. (Strictly speaking, the problem says $\alpha < 1$, but this check is instructive.) When $\alpha = 1$, the voltage drop across R_3 is $v_x - v_x = 0$V. This means no current flows through R_3, and we may ignore R_3 and the dependent source's branch. This in turn leaves just i_s and $R_1 + R_2$, which is a Norton equivalent. The Thevenin equivalent would have $v_{Th} = i_s(R_1 + R_2)$ and $R_{Th} = R_1 + R_2$. Our answer to (a) gives

$$v_{Th} = \frac{i_s (R_1 + R_2) \cancel{R_3}}{\cancel{R_3} + \cancel{(1-1)(R_1+R_2)}} = i_s (R_1 + R_2) \quad ✓$$

$$R_{Th} = \frac{(R_1 + R_2)\cancel{R_3}}{\cancel{R_3} + \cancel{(1-1)(R_1+R_2)}} = R_1 + R_2$$

More consistency checks are possible, including $R_3 = 0\,\Omega$, which causes dependent source αv_x to be in parallel with v_x. The solution in this case is $v_x = 0\,V$ (when $\alpha \neq 1$). This in turn means that the circuit acts like a wire, so $v_{Th} = 0\,V$ and $R_{Th} = 0\,\Omega$. The answer to (a) gives $V_{Th} = 0V$ and $R_{Th} = 0\,\Omega$ when $R_3 = 0\,\Omega$.

c) It is always the case that $i_N = \frac{v_{Th}}{R_{Th}}$ and $R_N = R_{Th}$. We find $i_N = i_s = \frac{v_{Th}}{R_{Th}}$ from (a).



$$R_N = R_{Th} = \frac{(R_1 + R_2)R_3}{R_3 + (1-\alpha)(R_1+R_2)}$$