Ex:

The voltage source in the above circuit is off for \( t > 0 \).

a) Find a symbolic expression for the Laplace-transformed output, \( V_o(s) \), in terms of not more than \( R_1, R_2, R_3, L, C \), and values of sources or constants.

b) Choose a numerical value for \( R_2 \) to make

\[
v_1(t) = v_m e^{-\alpha t} \cos(\beta t)
\]

where \( v_m, \alpha, \) and \( \beta \) are real-valued constants.

**Sol’n: a)** No current flows into the op-amp inputs. Thus, current flowing toward the - input from the left will flow through \( R_3 \) and into the op-amp then into the + or - power supply connections to the op-amp [which are not shown] then through the + or - power supply and back to the reference on the bottom rail [which was not shown].

Since the op-amp has negative feedback, we expect that \( v_- = v_+ \) at the op-amp inputs. In other words, \( v_- = v_+ = 0 \text{V} \), and we have a virtual ground (or reference) at the - input of the op-amp.

We can find \( V_o(s) \) from current, \( I(s) \), flowing toward the - input of the op-amp.
we have \( V_o(s) = -\Pi(s) R_3 \).

To find \( \Pi(s) \), we may treat the - input as reference.

First, however, we find initial conditions for the \( L \) and \( C \).

\( t=0^- \): \( V_C(t) = 6V \), \( C = \text{open} \), \( L = \text{wire} \)

We move to \( t>0 \) and include initial conditions on \( L \). \( V_L(t) = 0V = \text{wire for } t>0 \).

We have \( \Pi(s) = -\frac{6V}{s} \frac{1}{sL+R_1+R_2+\frac{1}{sC}} \)
or \[ \Pi(s) = \frac{-6V/L}{s^2 + \frac{R_1+R_2}{L} s + \frac{1}{LC}} \]

So \[ V_o(s) = -\Pi(s)R_3 \]

\[ V_o(s) = \frac{6V \cdot R_3/L}{s^2 + \frac{R_1+R_2}{L} s + \frac{1}{LC}} \]

b) \[ V_1(s) \] is the same as the \( V \) - drop across \( L \) and \( R_2 \).

\[ V_1(s) = \Pi(s) (sL + R_2) \]

or \[ V_1(s) = \frac{(6V/L)(sL+R_2)}{s^2 + \frac{R_1+R_2}{L} s + \frac{1}{LC}} \]

or \[ V_1(s) = -6V \frac{s + R_2/L}{s^2 + \frac{R_1+R_2}{L} s + \frac{1}{LC}} \]

From the form of \( v_1(t) \) given in the problem, we have another form for \( V_1(s) \):

\[ V_1(s) = \mathcal{L}\{v_m e^{\alpha t} \cos(\beta t)\} \]

\[ V_1(s) = v_m \frac{s + \alpha}{(s + \alpha)^2 + \beta^2} \]

\[ V_1(s) = v_m \frac{s + \alpha}{s^2 + 2\alpha s + \alpha^2 + \beta^2} \]

Matching coefficients of the powers of \( s \) in the two forms of \( V_1(s) \), we have the following equations:

\[ R_2/L = \alpha, \quad \frac{R_1+R_2}{L} = 2\alpha, \quad \frac{1}{LC} = \alpha^2 + \beta^2 \]
We have \( \frac{R_2}{\ell} = \alpha = \frac{R_1 + R_2}{2\ell} \).

The solution is \( R_2 = R_1 = 2 \, k\Omega \).

\[ R_2 = 2 \, k\Omega \]