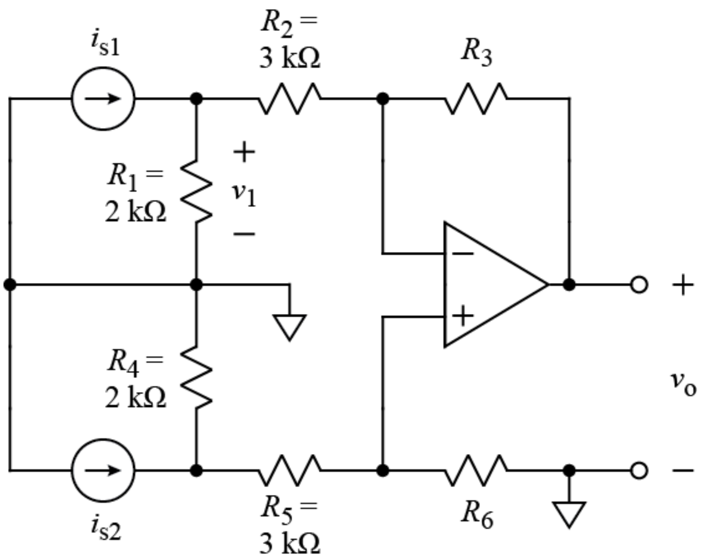
Ex:



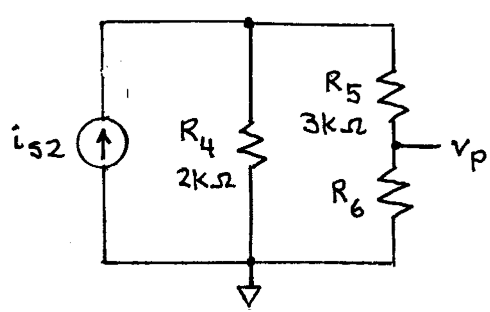
a) The above circuit operates in linear mode. Derive a symbolic expression for v_o. The expression must contain not more than the parameters i_{s1}, i_{s2}, R_1, R_2, R_3, R_4, R_5, and R_6.

b) Assuming $R_1 = R_4$, $R_2 = R_5$, and $R_3 = R_6$ derive a symbolic expression for v_o in terms of common mode and differential input voltages:

$$i_{cm} \equiv \frac{(i_{s2} + i_{s1})}{2} \qquad \text{and} \qquad i_{dm} \equiv i_{s2} - i_{s1}$$

The expression must contain not more than the parameters i_{cm}, i_{dm}, R_1, R_2, and R_3. Write the expression as i_{cm} times a term plus i_{dm} times a term.

soln: a) First, we find the voltage, v_p, at the + input of the op-amp. The circuit for finding v_p may be drawn as shown below.



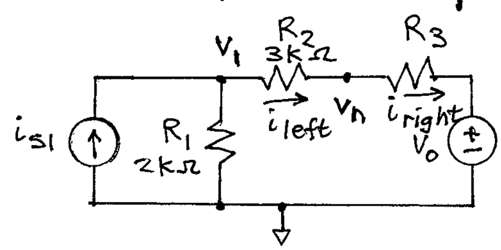
Using the i-divider formula, we find the current flowing through R_5 and R_6. We multiply this current by R_6 to get v_p:

$$v_p = i_{s2} \frac{R_4}{R_4 + R_5 + R_6} \cdot R_6$$

Second, we set the voltage, v_n, at the − input of the op-amp equal to v_p.

$$\boxed{v_n = v_p = i_{s2} \frac{R_4 R_6}{R_4 + R_5 + R_6}}$$

Third, we find an expression for the current i_{left} flowing toward the − input of the op-amp from the left. (This current is the current in R_2 measured with the arrow pointing to the right.) We also make sure that we use v_n in the expression for i_{left}.



Using node-voltage to find v_1, we have

$$-i_{s1} + \frac{v_1}{R_1} + \frac{v_1 - v_n}{R_2} = 0A$$

or

$$v_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = i_{s1} + \frac{v_n}{R_2}$$

or

$$v_1 = \left(i_{s1} + \frac{v_n}{R_2}\right) R_1 \| R_2$$

The value of i_{left} will be

$$i_{left} = \frac{v_1 - v_n}{R_2} = \left(i_{s1} + \frac{v_n}{R_2}\right)\frac{R_1 \| R_2}{R_2} - \frac{v_n}{R_2}$$

$$= i_{s1}\frac{R_1}{R_1+R_2} + \frac{v_n}{R_2}\left(\frac{R_1}{R_1+R_2} - 1\right)$$

$$= i_{s1}\frac{R_1}{R_1+R_2} + \frac{v_n}{R_2}\left(\frac{R_1}{R_1+R_2} - \frac{R_1+R_2}{R_1+R_2}\right)$$

$$= i_{s1}\frac{R_1}{R_1+R_2} - \frac{v_n}{R_1+R_2}$$

Note: We could obtain the same result by converting i_{s1} and R_1 into a Thevenin equivalent with voltage $i_{s1} R_1$ and resistance R_1. The current is then obtained directly as the above formula.

Note: We could also obtain the same result by treating v_n as a source voltage and using superposition of sources i_{s1} and v_n. We would obtain $i_{left1} = i_{s1}\frac{R_1}{R_1+R_2}$ (i-divider)

$+ \; i_{left2} = -\frac{v_n}{R_1+R_2}$ (i in R_2)

Fourth, we find an expression for the current, i_{right}, flowing to the right in R_3.

We make sure we use v_n and v_o in this expression.

$$i_{right} = \frac{v_n - v_o}{R_3}$$

Fifth, we equate i_{left} and i_{right} and solve for v_o.

$$i_{left} = \frac{i_{s1} R_1 - v_n}{R_1 + R_2} = \frac{v_n - v_o}{R_3} = i_{right}$$

or

$$v_o = -(i_{s1} R_1 - v_n) \frac{R_3}{R_1 + R_2} + v_n$$

or

$$v_o = v_n \frac{R_1 + R_2}{R_1 + R_2} + v_n \frac{R_3}{R_1 + R_2} - i_{s1} \frac{R_1 R_3}{R_1 + R_2}$$

or

$$\boxed{v_o = v_n \frac{R_1 + R_2 + R_3}{R_1 + R_2} - i_{s1} \frac{R_1 R_3}{R_1 + R_2}}$$

(where v_n formula is given earlier)

b)

we have $v_n = i_{s2} \dfrac{R_1 R_3}{R_1 + R_2 + R_3}$

and $$v_o = i_{s2} \frac{R_1 R_3}{R_1 + R_2} - i_{s1} \frac{R_1 R_3}{R_1 + R_2}$$

or

$$v_o = (i_{s2} - i_{s1}) \frac{R_1 R_3}{R_1 + R_2}$$

Note that we obtain the above before we even begin substituting for i_{cm} and i_{dm}.

Since $i_{dm} = i_{s2} - i_{s1}$, we have

$$v_o = i_{dm} \frac{R_1 R_3}{R_1 + R_2}$$