Given the resistor connected as shown and using not more than one each $R$, $L$, and $C$ in the dashed-line box, design a circuit to go in the dashed-line box that will produce the **band-pass** $|H(j\omega)|$ vs. $\omega$ shown above. That is:

\[
\max_{\omega} |H(j\omega)| = \frac{1}{4} \quad \text{and occurs at } \omega_0 = 10 \text{ M r/s}
\]

The bandwidth, $\beta$, of the filter is 500k r/s.

\[
|H(j\omega)| = 0 \quad \text{at } \omega = 0 \quad \text{and} \quad \lim_{\omega \to \infty} |H(j\omega)| = 0
\]

**Sol'n:** To achieve the peak at $\omega_0$, we may use a series LC in the top rail or a parallel LC from the top to bottom rail. To achieve a gain of $1/4$ at $\omega_0$, we must use a vertical resistance to form a V-divider with $R$.
We have two possible circuit configurations:

Series configuration

Parallel configuration

Note: \( R_1 \) must be to the right of \( L \) and \( C \) in order to have any effect from the \( L \) and \( C \) in the series configuration.

For the series configuration, the \( L \) and \( C \) are to act like a wire at \( \omega_0 = 10 \text{ M rad/s} \).

\[
j\omega_0 L + \frac{1}{j\omega_0 C} = 0 \quad \text{or} \quad \omega_0^2 = \left(10 \text{ M rad/s}\right)^2 = \frac{1}{LC}
\]

The bandwidth when using a series \( L \) and \( C \) is

\[
\beta = \frac{R_{eq}}{L} = 500 \text{ kHz}
\]

To determine the value of \( R_{eq} \), we view the filter as a standard RLC filter and a V-divider.

\[
H(j\omega) = \frac{V_o'}{V_i} \quad H'(j\omega) = H(j\omega) \cdot \frac{R_1}{R + R_1}
\]
The cutoff frequencies for $H(j\omega)$ are the same as the cutoff frequencies for $H'(j\omega)$:

$$
\omega_{c1,2} = \pm \frac{R_{eq} \pm \sqrt{(R_{eq})^2 + \omega_0^2}}{2L}, \quad \beta = \frac{R_{eq}}{L}
$$

where $R_{eq} = R + R_1 = 400 \Omega$.

Using $\beta = \frac{R_{eq}}{L}$, we find $L$:

$$
L = \frac{R_{eq}}{\beta} = \frac{400 \Omega}{500 \text{kr} \cdot \text{s}} = 0.8 \text{ mH or } 800 \text{ } \mu\text{H}
$$

Using $\omega_0^2 = \frac{1}{LC}$ and $L = 800 \text{ } \mu\text{H}$, we find $C$:

$$
C = \frac{1}{\omega_0^2 L} = \frac{1}{10^4 \cdot 10^6 \cdot 800 \text{ } \mu\text{F}} = \frac{1 \text{ } \mu\text{F}}{80 \text{ } \text{K}}
$$

$$
C = 12.5 \text{ } \mu\text{F}
$$

Summary of series RLC: $R_1 = 100 \Omega$, $L = 800 \text{ } \mu\text{H}$, $C = 12.5 \text{ } \mu\text{F}$

For the parallel configuration, we move $R_1$ to the left of $L$ and $C$ and use a Thévenin equivalent of $V_i$, $R$, and $R_1$. 

![Parallel Configuration Diagram]

Where $V_{10} = 500 \text{ } \text{V}$.
To find the Thévenin equivalent, we find \( V_{\text{Th}} \) by finding the open-circuit of the \( V_i, R_j \) and \( R_k \) circuit.

\[
\begin{align*}
V_i \quad & \begin{array}{c}
\oplus \\
\downarrow
\end{array} \quad R_j \quad \begin{array}{c}
\oplus \\
\downarrow
\end{array} \\
= & \quad \begin{array}{c}
\oplus \\
\downarrow
\end{array} \quad R_k \quad \begin{array}{c}
\oplus \\
\downarrow
\end{array} \quad 100 \Omega \\
& \quad \begin{array}{c}
\oplus \\
\downarrow
\end{array} \\
& a \\
& b
\end{align*}
\]

\[
V_{\text{Th}} = \frac{V_i \cdot R_j}{R_j + R_k} = \frac{V_i}{4}
\]

To find \( R_{\text{Th}} \), we turn off \( V_i \) and look in from terminals a and b. The resistance seen is

\[
R_{\text{Th}} = R \parallel R_j \parallel 300 \Omega \parallel 100 \Omega = 75 \Omega
\]

Using the filter with the Thévenin equivalent, we have

\[
H(j\omega) = \frac{V_o}{V_i} = \frac{R_k}{R_j + R_k} \quad \frac{V_o}{V_i} = \frac{1}{4} H'(j\omega)
\]

where \( H'(j\omega) = \frac{V_o}{V_{i'}} \) where \( V_{i'} = \frac{R_k}{R_j + R_k} V_i \)

The cutoff frequencies of \( H'(j\omega) \) are the same as the cutoff frequencies of \( H(j\omega) \).

\[
\omega_{c1,2} = \pm \frac{1}{\sqrt{\frac{2RC}{T_h}}} \sqrt{\left(\frac{1}{2RC}\right)^2 + \omega_0^2}, \quad \beta = \frac{1}{\sqrt{RC}}
\]

\[
\omega_0 = \frac{1}{\sqrt{LC}}
\]

Using \( R_{\text{Th}} \) and \( \beta \), we find \( C \):

\[
C = \frac{1}{R_{\text{Th}}^2} = \frac{1}{75 \Omega \cdot 500k \Omega} = 26.6 \text{ nF}
\]

Using \( L \) and \( \omega_0^2 \), we find \( L \):

\[
L = \frac{1}{\omega_0^2 C} = \frac{1}{10M10M 26.6n} = 375 \text{ nH}
\]