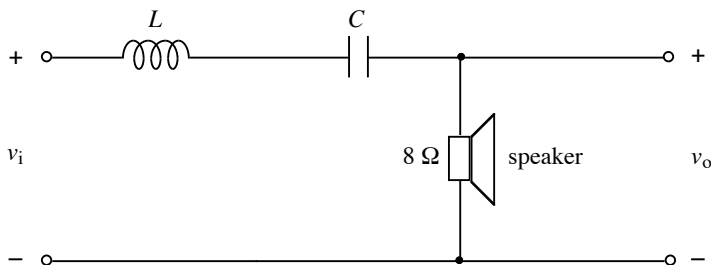


Ex:



The above circuit is part of a simple crossover network for driving a midrange speaker having an impedance of 8Ω . The circuit is described at the following web site: <http://www.termpro.com/articles/xover2.html>. A more in-depth discussion of crossover networks may be found at <http://sound.westhost.com/lr-passive.htm>.

- The web site describing the above bandpass filter suggests using cutoff frequencies of $f_{C1} = 130$ Hz and $f_{C2} = 4$ kHz. Determine the L and C values that yield these cutoff frequencies.
- Plot $|V_o/V_i|$ versus ω .

SOL'N: a) This is a standard band-pass filter. The transfer function is a voltage-divider divided by the input voltage:

$$H(j\omega) \equiv \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{R}{R + j\omega L - \frac{j}{\omega C}} = \frac{1}{1 + j\frac{1}{R}\left(\omega L - \frac{1}{\omega C}\right)}$$

To calculate the cutoff frequencies, we set the denominator equal to $1 \pm j$, which has a magnitude of $\sqrt{2}$. This is equivalent to the following equation:

$$\frac{1}{R}\left(\omega L - \frac{1}{\omega C}\right) = \pm 1$$

After multiplying by $\omega R/L$, we obtain two quadratic equations with four roots. Taking the positive roots, we obtain the cutoff frequencies for this filter as follows:

$$\omega_{C1,2} = \pm \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

where $R = 8\Omega$.

The following observations simplify our calculations for values of L and C that give the desired cutoff frequencies:

$$\omega_{C2} - \omega_{C1} = \left(\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \right) - \left(-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \right)$$

or

$$\omega_{C2} - \omega_{C1} = \frac{R}{L}$$

or

$$L = \frac{R}{\omega_{C2} - \omega_{C1}}$$

Also,

$$\omega_{C1}\omega_{C2} = \left(-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \right) \left(\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \right)$$

or

$$\omega_{C1}\omega_{C2} = -\left(\frac{R}{2L}\right)^2 + \left(\sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right)^2 = \frac{1}{LC} = \omega_0^2$$

or

$$C = \frac{1}{\omega_{C1}\omega_{C2}L}$$

Now we compute the cutoff frequencies in r/s:

$$\omega_{C1} = 2\pi f_{C1} = 2\pi(130) \text{ r/s} = 817 \text{ r/s}$$

$$\omega_{C2} = 2\pi f_{C2} = 2\pi(4 \text{ k}) \text{ r/s} = 25.1 \text{ kr/s}$$

Using our formulas from above yields the following:

$$L = \frac{R}{\omega_{C2} - \omega_{C1}} = \frac{8}{25.1\text{k} - 0.817\text{k}} \approx 330 \mu\text{H}$$

and

$$C = \frac{1}{\omega_{C1}\omega_{C2}L} = \frac{1}{0.817\text{k} \cdot 25.1\text{k} \cdot 330\mu} \text{F} \approx 150 \mu\text{F}$$

b) The circuit is a voltage divider:

$$H(j\omega) = \frac{V_o}{V_i} = \frac{R}{R + j\omega L + 1/j\omega C} = \frac{1}{1 + j\frac{1}{R}(\omega L - 1/\omega C)}$$

We use the following Matlab code to plot the frequency response:

```
% ECE2260F07_HW3p3Matlab.m
%
% Plot of filter's frequency response curve

figure(1)
omega = 1:30e1:30e3;
s = j * omega;

FilterResp = 1./(1 + j * (1/8)*(omega*330e-6 - 1
./ (omega*150e-6)));

plot(omega,abs(FilterResp))
axis([0, max(omega), 0, 1])
xlabel('omega')
ylabel('|H|')
```

