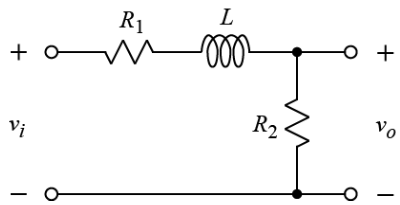
HOMEWORK #19 prob 2 solution

Ex:



$R_1 = 150\ \Omega$ $R_2 = 750\ \Omega$ $L = 1\ \mu\text{H}$

a) Determine the transfer function V_o/V_i. **Hint:** switch the order of R_1 and L and use a voltage divider.

b) Express the maximum of $|V_o / V_i|$ as a function of R_1 and R_2.

sol'n: a) If we ignore the hint, we have the following calculation:

$$H(j\omega) = \frac{V_o}{V_i} = \frac{\cancel{V_i}}{\cancel{V_i}} \frac{R_2}{R_1 + R_2 + j\omega L}$$

Putting this in the form $H(j\omega) = k \dfrac{1}{1 \pm jX}$ is achieved by factoring out R_2 from the numerator and $R_1 + R_2$ from the denominator.

$$H(j\omega) = \frac{R_2}{R_1 + R_2} \cdot \frac{1}{1 + j\dfrac{\omega L}{R_1 + R_2}}$$

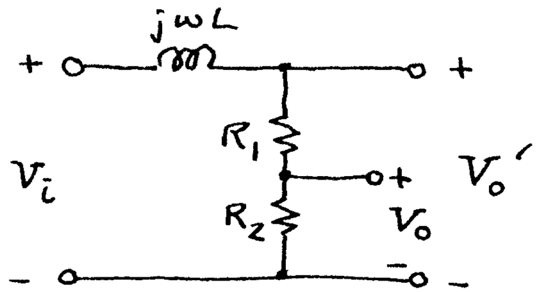
So $k = \dfrac{R_2}{R_1 + R_2}$ and $X = \dfrac{\omega L}{R_1 + R_2}$.

$$k = \frac{750\ \Omega}{150 + 750\ \Omega} = \frac{5}{6}$$

$$X = \frac{\omega\ 1\mu}{150 + 750\ \Omega} = \frac{\omega}{900\ \text{Mr/s}}$$

$$H(j\omega) = \frac{5}{6} \frac{1}{1 + j\dfrac{\omega}{900\ \mathrm{Mr/s}}}$$

If we use the hint, we view the filter as an RL filter, (with $R = R_1 + R_2$), and a V divider:



We have filter transfer function:

$$H'(j\omega) \equiv \frac{V_o'}{V_i} = \frac{R_1 + R_2}{R_1 + R_2 + j\omega L} = \frac{1}{1 + j\omega \dfrac{L}{R_1 + R_2}}$$

We multiply this by a voltage-divider gain term to our desired $H(j\omega)$:

$$V_o = V_o' \cdot \frac{R_2}{R_1 + R_2}$$

or $$\frac{V_o}{V_o'} = \frac{R_2}{R_1 + R_2}$$

So $$H(j\omega) = \frac{V_o}{V_i} = \frac{V_o'}{V_i} \cdot \frac{V_o}{V_o'}$$

or $$H(j\omega) = \frac{R_2}{R_1 + R_2} \cdot \frac{1}{1 + j\omega \dfrac{L}{R_1 + R_2}}$$

$$H(j\omega) = \frac{5}{6} \frac{1}{1 + j\frac{\omega}{900\ \text{Mr/s}}}$$

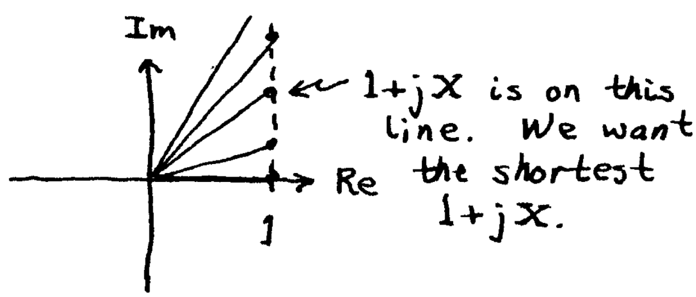
b) $$\max_{\omega} \left| \frac{V_o}{V_i} \right| \equiv \max_{\omega} \left| H(j\omega) \right|$$

$$= \frac{5}{6} \max_{\omega} \left| \frac{1}{1 + j\frac{\omega}{900\ \text{Mr/s}}} \right|$$

$$= \frac{5}{6} \max_{\omega} \frac{1}{\left| 1 + j\frac{\omega}{900\ \text{Mr/s}} \right|}$$

$$= \frac{5}{6} \frac{1}{\min_{\omega} \left| 1 + j\underbrace{\frac{\omega}{900\ \text{Mr/s}}}_{X} \right|}$$

We observe that X ranges from 0 to ∞ as ω ranges from 0 to ∞.



The shortest $1+jX$ is $1+j0$, with $|1+j0| = 1$.

So $$\max_{\omega} \left| \frac{V_o}{V_i} \right| = \frac{5}{6} \frac{1}{1} = \frac{5}{6} = \frac{R_2}{R_1 + R_2}$$