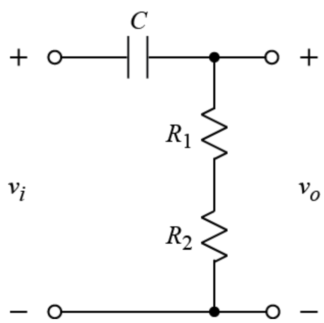
HOMEWORK #19 prob 1 solution

Ex:



$R_1 = 120\ \text{k}\Omega \qquad R_2 = 130\ \text{k}\Omega \qquad C = 200\ \text{nF}$

a) Determine the transfer function V_o/V_i.

b) Find ω such that $|V_o / V_i| = 1/\sqrt{2}$.

c) Find ω such that $\angle V_o / V_i = 45°$.

d) Is it true that $\left|\frac{1}{j\omega C}\right| = |R_1 + R_2|$ at $\omega = \omega_C$?

sol'n: a) We need only combine R_1 and R_2 and treat this as a standard RC filter.

$$V_o = V_i \frac{R_1 + R_2}{R_1 + R_2 + \frac{1}{j\omega C}} \qquad \text{V-divider}$$

$$H(j\omega) = \frac{V_o}{V_i} = \frac{R_1 + R_2}{R_1 + R_2 + \frac{1}{j\omega C}}$$

It is convenient, when finding cutoff frequencies, to write $H(j\omega)$ in the following form:

$$H(j\omega) = k \frac{1}{1 + jX}$$ where k is real and constant and X is real

We obtain the desired form by dividing the top and bottom by R_1+R_2.

$$H(j\omega) = \frac{1}{1 + \frac{1}{j\omega(R_1+R_2)C}} = \frac{1}{1 + \frac{1}{j\omega\, 250k\, 200n}\ r/s}$$

$$H(j\omega) = \frac{1}{1 - j\frac{1}{\omega(R_1+R_2)C}} = \frac{1}{1 - j\frac{20}{\omega}\ r/s}$$

b) To obtain $\left|\frac{V_o}{V_i}\right| = \frac{1}{\sqrt{2}}$ from the above formula, we observe that $|1 \pm j| = \sqrt{2}$, (i.e., $|1 \pm j| = \sqrt{1^2 + 1^2}$

Here, we have $H(j\omega) = \frac{1}{1 - jX}$

where $X = \frac{1}{\omega(R_1+R_2)C}$.

the sol'n we seek is where

$$X = 1.$$

Thus, $\frac{1}{\omega(R_1+R_2)C} = 1$,

or $\omega = \frac{1}{(R_1+R_2)C} = 20\, r/s$

This is just the calculation of the cutoff frequency.

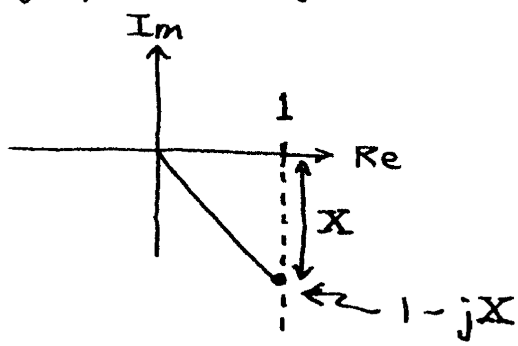
c) $\angle(V_o/V_i) = \angle V_o - \angle V_i$ always.

Here $\angle\left(\frac{V_o}{V_i}\right) = \angle H(j\omega) = \angle 1 - \angle 1 - jX$

where $X = \frac{1}{\omega(R_1+R_2)C}$ is real.

Now, $\angle 1 = 0°$ since 1 is a real number.

To understand $\angle 1 - jX$, we may use a graph of $1 - jX$:



We want $\angle H(j\omega) = 45° = 0° - \angle(1 - jX)$

So we want $\angle(1 - jX) = -45°$.

Thus, we want $X = 1$ since

$\angle 1 - j = -45°$.

So $X \equiv \frac{1}{\omega(R_1+R_2)C} = 1$

This is the same problem as in (b).

$\omega = \frac{1}{(R_1+R_2)C}$, (which is cutoff freq)

$\omega = 20$ r/s

d) $$\left|\frac{1}{j\omega C}\right| = \frac{1}{|j\omega C|} = \frac{1}{\omega C}$$

$$|R_1 + R_2| = R_1 + R_2$$

So $$\frac{1}{\omega C} = R_1 + R_2$$

or $$\omega = \frac{1}{(R_1+R_2)C} = 20 \text{ r/s}$$

This is indeed the same as ω_c.