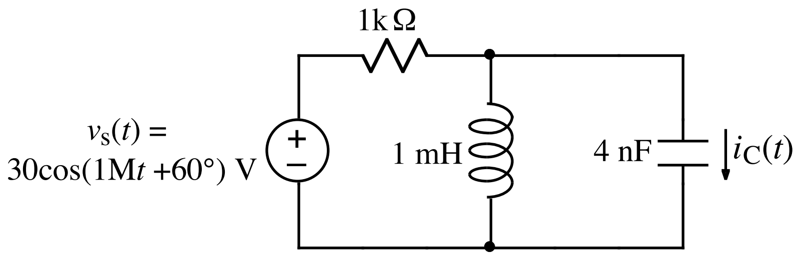
Ex:



a) Find the phasor value for $v_S(t)$.

b) Draw the frequency-domain circuit diagram, including the phasor value for $v_S(t)$ and impedance values for components.

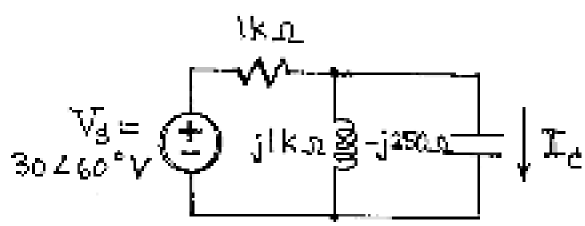
c) Find the phasor value for $i_C(t)$.

soln: a) $\mathcal{P}[30 \cos(1Mt + 60°)\ V] = 30 e^{j60°}$ V or $30 \angle 60°$ V

b) We calculate the impedances:

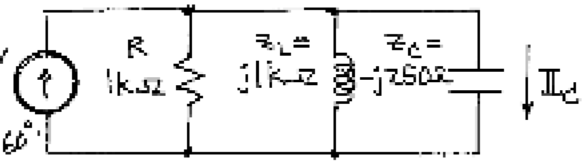
$$Z_L = j\omega L = j\, 1M \cdot 1\, mH = j1k\Omega$$

$$Z_C = \frac{1}{j\omega C} = \frac{-j}{\omega C} = \frac{-j}{1M\, 4nF} = \frac{-j}{4m}\Omega = -j250\Omega$$



c) One way to solve this problem is to use a Norton equivalent for V_s and the 1kΩ resistor.

$$\mathbb{I} = \frac{30\angle 60^\circ \text{ V}}{1\text{k}\Omega} = 30\text{mA}\angle 60^\circ$$



Using the current divider formula, we have

$$\mathbb{I}_C = \mathbb{I}\,\frac{R \| Z_L}{R \| Z_L + Z_C}$$

$$= \mathbb{I}\,\frac{1}{1 + \dfrac{Z_C}{R \| Z_L}}$$

$$= \mathbb{I}\,\frac{1}{1 + Z_C\left(\dfrac{1}{R} + \dfrac{1}{Z_L}\right)}$$

$$= \mathbb{I}\,\frac{1}{1 + \dfrac{Z_C}{R} + \dfrac{Z_C}{Z_L}}$$

$$= 30\text{mA}\angle 60^\circ\,\frac{1}{1 + \dfrac{-j250\Omega}{1\text{k}\Omega} + \dfrac{-j250\Omega}{j1\text{k}\Omega}}$$

$$= 30\text{mA}\angle 60^\circ\,\frac{1}{1 - \frac{1}{4} - j\frac{1}{4}}$$

$$= \frac{30\text{mA}\angle 60^\circ}{\frac{3}{4} - j\frac{1}{4}}$$

$$= \frac{120\text{ mA}\angle 60^\circ}{3-j}$$

$$= \frac{120\text{ mA}\angle 60^\circ}{3-j} \quad \frac{3+j}{3+j}$$

$$= \frac{120\text{ mA}\angle 60^\circ\ (3+j)}{3^2+1^2}$$

$$= 12\text{ mA}\angle 60^\circ\ (3+j)$$

$$= 12\text{ mA}\left(\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)(3+j)$$

$$= 12\text{ mA}\left[\frac{3}{2}-\frac{\sqrt{3}}{2}+j\left(\frac{1}{2}+\frac{3\sqrt{3}}{2}\right)\right]$$

$$\mathbb{I}_c = 6\text{ mA}\left[3-\sqrt{3}+j\left(1+3\sqrt{3}\right)\right]$$

Note: This is an exact answer, but an approximate polar answer is more useful.

$$\mathbb{I}_c = 12\text{ mA}\angle 60^\circ\ (3+j)$$

$$= 12\text{ mA}\angle 60^\circ \cdot \sqrt{3^2+1^2}\ \angle \tan^{-1}\frac{1}{3}$$

$$= 12\text{ mA}\angle 60^\circ \cdot \sqrt{10}\ \angle 18.4^\circ$$

$$\mathbb{I}_c = 12\sqrt{10}\text{ mA}\angle 78.4^\circ$$

Note: $i_c(t) = 12\sqrt{10}\text{ mA}\cos(1Mt + 78.4^\circ)$