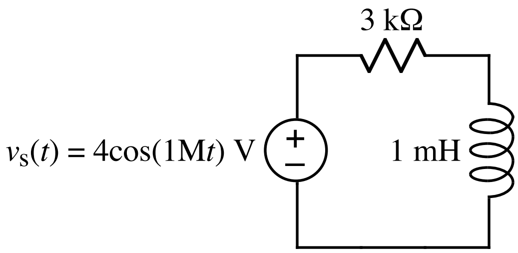
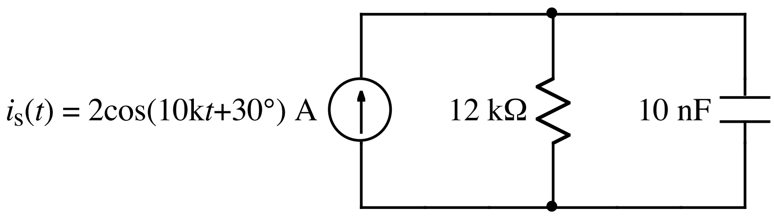
HOMEWORK #17 prob 4 solution

Ex:



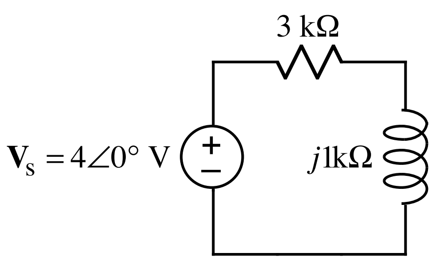
a) Find time-domain expressions for the waveforms of the voltages across the R and L in the above circuit.



b) Find time-domain expressions for the waveforms of the currents through the R and C in the above circuit.

SOL'N: a) First, we transform the circuit to the frequency-domain.

$$j\omega L = j1\text{M} \cdot 1\text{m } \Omega = j1 \text{ k}\Omega$$



Second, we use a voltage-divider formula to find the voltage across the R and L.

$$\mathbf{V}_R = 4\angle 0°\text{V}\frac{3\text{k}\Omega}{3\text{k}\Omega + j1\text{k}\Omega} = 4\text{V}\frac{3}{3+j1} = 4\text{V}\frac{3}{3+j}\frac{3-j}{3-j}$$

or

$$\mathbf{V}_R = 12\text{V}\frac{3-j}{10} = 1.2\text{V} \cdot \sqrt{3^2+1^2}\angle\tan^{-1}\left(\frac{-1}{3}\right) = 1.2\sqrt{10}\angle -18.4° \text{ V}$$

or

$$\mathbf{V}_R = 3.79\angle -18.4° \text{ V}$$

The calculation for the inductor voltage is similar to the above.

$$\mathbf{V}_{\mathrm{L}} = 4\angle 0°\mathrm{V}\frac{j1\mathrm{k}\Omega}{3\mathrm{k}\Omega + j1\mathrm{k}\Omega} = 4\mathrm{V}\frac{j1}{3+j1} = 4\mathrm{V}\frac{j}{3+j}\frac{3-j}{3-j}$$

or

$$\mathbf{V}_{\mathrm{L}} = 4\mathrm{V}\frac{1+j3}{10} = 0.4\mathrm{V}\cdot\sqrt{1^2+3^2}\angle\tan^{-1}\left(\frac{3}{1}\right) = 0.4\sqrt{10}\angle 71.6°\ \mathrm{V}$$

or

$$\mathbf{V}_{\mathrm{L}} = 1.26\angle 71.6°\ \mathrm{V}$$

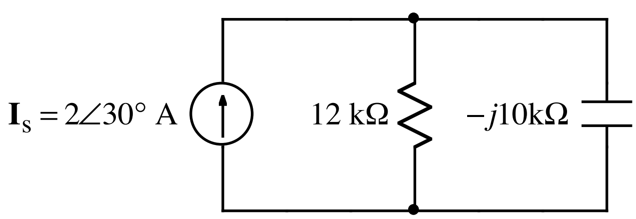
Third, we take the inverse phasor.

$$v_{\mathrm{R}}(t) = 1.2\sqrt{10}\cos(1\mathrm{M}t - 18.4°)\ \mathrm{V}$$

$$v_{\mathrm{L}}(t) = 0.4\sqrt{10}\cos(1\mathrm{M}t + 71.6°)\ \mathrm{V}$$

b) First, we transform the circuit to the frequency-domain.

$$\frac{1}{j\omega C} = \frac{1}{j10\mathrm{k}\cdot 10\mathrm{n}}\Omega = -j10\mathrm{k}\Omega$$



Second, we use a current-divider formula to find the current through the *R* and *C*.

$$\mathbf{I}_{\mathrm{R}} = 2\angle 30°\mathrm{A}\frac{-\mathrm{j}10\mathrm{k}\Omega}{12\mathrm{k}\Omega - j10\mathrm{k}\Omega} = 2\angle 30°\mathrm{A}\frac{10\angle -90°}{\sqrt{12^2+10^2}\angle\tan^{-1}\left(\frac{-10}{12}\right)}\ \mathrm{A}$$

or

$$\mathbf{I}_{\mathrm{R}} = \frac{10}{\sqrt{61}}\angle 30° - 90° - \tan^{-1}\left(\frac{-10}{12}\right)\mathrm{A} = \frac{10}{\sqrt{61}}\angle -20.2°\ \mathrm{A}$$

The calculation for the capacitor current is similar to the above.

$$\mathbf{I}_C = 2\angle 30°\,\text{A}\,\frac{12\text{k}\Omega}{12\text{k}\Omega - j10\text{k}\Omega} = 2\angle 30°\,\text{A}\,\frac{12}{\sqrt{12^2+10^2}\angle \tan^{-1}\left(\frac{-10}{12}\right)}\ \text{A}$$

or

$$\mathbf{I}_C = \frac{12}{\sqrt{61}}\angle 30° - \tan^{-1}\left(\frac{-10}{12}\right)\text{A} = \frac{10}{\sqrt{61}}\angle 69.8°\ \text{A}$$

Third, we take the inverse phasor.

$$i_R(t) = \frac{10}{\sqrt{61}}\cos(10\text{k}t - 20.2°)\ \text{A}$$

$$i_C(t) = \frac{12}{\sqrt{61}}\cos(10\text{k}t + 69.8°)\ \text{A}$$