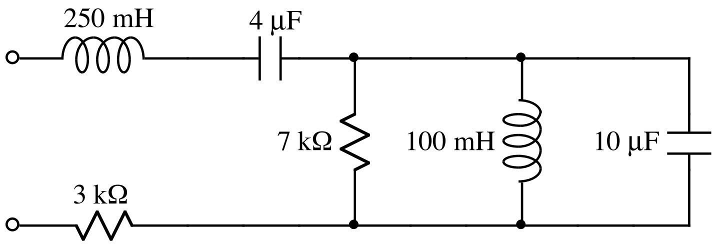
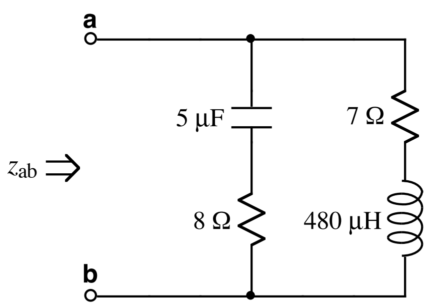
ECE 2240
F 12

HOMEWORK #17 prob 2 solution

Ex: a) Find the total impedance of the circuitry shown below if ω = 1000 rad/s.



b) Given ω = 50 k rad/s, find z_{ab}.



SOL'N: a) We convert to the frequency-domain by computing impedances.

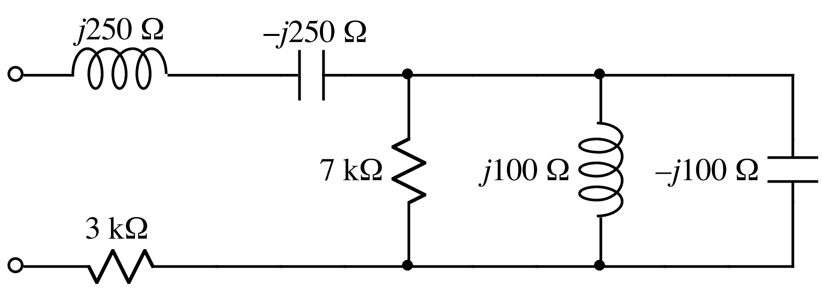
$$j\omega L = j1\text{k}\cdot 250\text{m}\ \Omega = j250\ \text{k}\Omega$$

$$\frac{1}{j\omega C} = \frac{1}{j1\text{k}\cdot 4\mu}\Omega = -j250\ \Omega$$

$$j\omega L = j1\text{k}\cdot 100\text{m}\ \Omega = j100\ \text{k}\Omega$$

$$\frac{1}{j\omega C} = \frac{1}{j1\text{k}\cdot 10\mu}\Omega = -j100\ \Omega$$

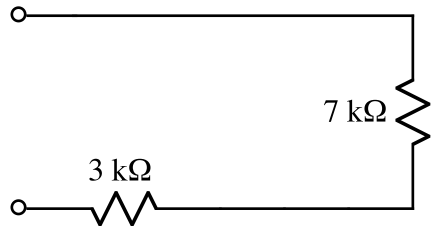
The circuit diagram in the frequency-domain is shown below.



The series *L* and *C* in series at the top left of the circuit sum to zero, which means they cancel out to act like a wire. The parallel *L* and *C* at the right combine to create an equivalent impedance of infinity, or an open circuit.

$$j100 \parallel -j100\ \Omega = j100\ \Omega \cdot 1 \parallel -1 = j100\ \Omega \cdot \frac{1(-1)}{1-1} = j100\ \Omega \cdot \frac{1}{0} = \infty\ \Omega$$

Thus, the L and C on the right disappear. We are left with a simple circuit consisting of only two resistors:



The equivalent impedance is obviously 10 kΩ.

$z_{\text{tot}} = 10\ \text{k}\Omega$

sol'n: We compute impedances using

$z_R = R, \quad z_L = j\omega L, \quad z_C = \frac{1}{j\omega C} = \frac{-j}{\omega C}$.

$\therefore z_L = j \cdot 50\text{k rad/s} \cdot 480\ \mu\text{H}$

$\quad " = j\ 50 \cdot 480\ \text{m}\Omega$

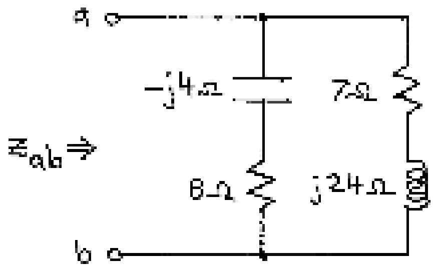
$\quad " = j\ 24\ \Omega$

$z_C = \frac{-j}{50\text{k rad/s} \cdot 5\ \mu\text{F}}$

$\quad = \frac{-j}{250\text{m}}\ \Omega$

$\quad = -j4\ \Omega$

Now we draw the frequency- (or s-domain) model:



We have $Z_{ab} = (8 - j4\ \Omega) \| (7 + j24\ \Omega)$

$$= \frac{(8-j4)(7+j24)}{8-j4+7+j24}\ \Omega$$

$$= \frac{\sqrt{8^2+4^2}\,\angle\tan^{-1}(-4/8)\ \sqrt{7^2+24^2}\,\angle\tan^{-1}(24/7)}{15+j20}\ \Omega$$

$$= \frac{4\sqrt{5}\angle -26.6^\circ \cdot 25\angle 73.7^\circ}{25\angle 53.1^\circ}\ \Omega$$

$$= 4\sqrt{5}\angle -26.6^\circ + 73.7^\circ - 53.1^\circ\ \Omega$$

$$Z_{ab} = 4\sqrt{5}\angle -6^\circ\ \Omega$$