HOMEWORK #16 prob 1 solution

Ex: a) Find the real part of $z = e^{j\pi/2}$.

b) Find the rectangular form of $e^{j\pi/2}$.

c) Find the rectangular form of $5\angle 25° \cdot 8\angle 35°$

d) Find the magnitude of $\left(\frac{j^3}{2+j4}\right)\left(\frac{30e^{j129°}}{2-j}\right)$.

e) Find the polar (magnitude and angle) form of $\sqrt{2+\sqrt{3}} - j\sqrt{2-\sqrt{3}}$

SOL'N: a) Use Euler's formula:

$$\text{Re}\left[e^{j\pi/2}\right] = \text{Re}[\cos\pi/2 + j\sin\pi/2] = \text{Re}[0+j] = 0$$

b) From the answer to (a), we have

$$e^{j\pi/2} = j.$$

c) We first multiply the numbers in polar form.

$$5\angle 25° \cdot 8\angle 35° = 5(8)\angle 25° + 35° = 40\angle 60° = 40e^{j60°}$$

Now we convert to rectangular form using Euler's formula.

$$5\angle 25° \cdot 8\angle 35° = 40\cos(60°) + j40\sin(60°) = 40 \cdot \frac{1}{2} + j40\frac{\sqrt{3}}{2}$$

or

$$5\angle 25° \cdot 8\angle 35° = 20 + j20\sqrt{3}$$

d) We take the magnitude of each term of a product or quotient. We must keep sums as is, however.

$$\left|\left(\frac{j^3}{2+j4}\right)\left(\frac{30e^{j129°}}{2-j}\right)\right| = \frac{\left|j^3\right|}{|2+j4|}\frac{\left|30e^{j129°}\right|}{|2-j|} = \frac{1^3 \cdot 30}{\sqrt{2^2+4^2}\sqrt{2^2+1^2}}$$

or

$$\left|\left(\frac{j^3}{2+j4}\right)\left(\frac{30e^{j129°}}{2-j}\right)\right| = \frac{30}{\sqrt{20}\sqrt{5}} = 3$$

e) We use the Pythagorean theorem to find the magnitude:

$$A = \sqrt{2+\sqrt{3}}^2 + \sqrt{2-\sqrt{3}}^2 = 2+\sqrt{3}+2-\sqrt{3} = 4$$

The tangent of the angle is the imaginary part over the real part.

$$\phi = \tan^{-1}\frac{\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}} = 15°$$

Our answer:

$$\sqrt{2+\sqrt{3}} - j\sqrt{2-\sqrt{3}} = 4\angle 15°$$