Ex: Use a Taylor series for e^x, cos(x), and sin(x) to shown the following equation is valid: (This is Euler's formula.)

$$e^{jx} = \cos x + j \sin x$$

SOL'N: Our Taylor series for the exponential and cosine and sine are

$$e^x = 1 + x + \frac{x^2}{2} + \ldots + \frac{x^n}{n!} + \ldots$$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \ldots + \frac{(-1)^n x^{2n}}{(2n)!} + \ldots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \ldots$$

When we substitute jx for x, we find the odd terms are imaginary and give the sine function, whereas the even terms are real and give the cosine function.

$$e^{jx} = 1 + jx + \frac{(jx)^2}{2} + \ldots + \frac{(jx)^n}{n!} + \ldots$$

or

$$e^{jx} = 1 + jx + \frac{j^2 x^2}{2} + \ldots + \frac{j^n x^n}{n!} + \ldots$$

or

$$e^{jx} = 1 + jx - \frac{x^2}{2} - j\frac{x^3}{3!} + \frac{x^4}{4!} + j\frac{x^5}{5!} - \ldots + \frac{j^n x^n}{n!} + \ldots$$

or

$$e^{jx} = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \ldots + \frac{j^{2n} x^{2n}}{(2n)!} + \ldots$$
$$+ jx - j\frac{x^3}{3!} + j\frac{x^5}{5!} - \ldots + \frac{j^{2n+1} x^{2n+1}}{(2n+1)!} + \ldots$$

or

$$e^{jx} = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \ldots + \frac{(-1)^n x^{2n}}{(2n)!} + \ldots$$

$$+ jx - j\frac{x^3}{3!} + j\frac{x^5}{5!} - \ldots + j\frac{(-1)^n x^{2n+1}}{(2n+1)!} + \ldots$$

or

$$e^{jx} = \cos x + j \sin x$$