Ex: Give numerical answers to each of the following questions:

a)   Rationalize $\dfrac{175 - j600}{-3 + j4}$. Express your answer in rectangular form.

b)   Find the magnitude of $\dfrac{1}{2} + j\dfrac{\sqrt{3}}{2}$.

c)   Find the real part of $\dfrac{(1 + j)^4}{1 + j\sqrt{3}}$.

SOL'N: a) To rationalize, we multiply the numerator and denominator by the conjugate of the denominator.

$$\frac{175 - j600}{-3 + j4} \cdot \frac{-3 - j4}{-3 - j4} = \frac{175(-3) - 600(4) - j175(4) - j600(-3)}{(-3)^2 + 4^2}$$

$$\frac{175 - j600}{-3 + j4} = \frac{-2925 + j1100}{25} = -117 + j44$$

b) We think of the complex number as a vector and find its length and its angle relative to the real axis.

$$\frac{1}{2} + j\frac{\sqrt{3}}{2} = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\, e^{j\tan^{-1}\frac{\sqrt{3}/2}{1/2}} = \sqrt{\frac{1}{4} + \frac{3}{4}}\, e^{j60°} = 1e^{j60°}$$

or

$$\frac{1}{2} + j\frac{\sqrt{3}}{2} = e^{j60°}$$

c)

$$\text{Re}\left[\frac{(1+j)^4}{1 + j\sqrt{3}}\right] = \text{Re}\left[\frac{\left(\sqrt{2}e^{j45°}\right)^4}{2e^{j60°}}\right] = \text{Re}\left[\frac{4e^{j180°}}{2e^{j60°}}\right] = \text{Re}\left[2e^{j(180° - 60°)}\right]$$

or

$$\text{Re}\left[\frac{(1+j)^4}{1 + j\sqrt{3}}\right] = \text{Re}\left[2e^{j120°}\right] = \text{Re}\left[2\cos(120°) + j2\sin(120°)\right]$$

or

$$\text{Re}\left[\frac{(1+j)^4}{1+j\sqrt{3}}\right] = 2\cos(120°) = 2\left(-\frac{1}{2}\right) = -1$$