Ex:

\[ i_g(t) = \begin{cases} 3 \text{ A} & t < 0 \\ 0 \text{ A} & t \geq 0 \end{cases} \]

a) Write the Laplace transform \( I_g(s) \) of \( i_g(t) \).

b) Write the Laplace transform \( V_o(s) \) of \( v_o(t) \). Be sure to include the effects of initial conditions, if they are nonzero.

c) Write a numerical time-domain expression for \( v_o(t) \) where \( t \geq 0 \).

**SOL'N:**

a) The Laplace transform depends only on what the input signal is from time 0 to \( \infty \).

\[ I_g(t) = \mathcal{L}\{i_g(t)\} = \mathcal{L}\{0\} \text{A} = 0 \text{ A} \]

b) We find initial conditions by considering circuit at \( t = 0^- \). The circuit has a 3 A input and has reached equilibrium. Thus, the \( L \) acts like a wire, shorting out the \( R \) and \( C \). Thus, the initial conditions on the \( C \) are zero and all of the input current flows through the \( L \).

\[ i_L(0^-) = 3 \text{ A} \]

Our circuit model includes initial conditions for the \( L \) but no input source, since the input source is zero:
The output voltage is found by using a voltage-divider formula. We observe that $V_o(s)$ is measured across the $R$ and $C$. Thus, we may use a voltage-divider formula that avoids the need to add the voltage source to our answer. (An alternative approach is to use a voltage-divider to find the voltage across the $L$ and then add the voltage source to our answer.)

$$V_o(s) = -1.5 \frac{R + \frac{1}{sC}}{sL + R + \frac{1}{sC}} = -1.5 \frac{25 + \frac{200}{s}}{\frac{s}{2} + 25 + \frac{200}{s}}$$

$$= -1.5 \frac{50s + 400}{s^2 + 50s + 400}$$

$$= -75 \frac{s + 8}{(s+10)(s+40)}$$

c) The output voltage versus time is the inverse Laplace transform of $V_o(s)$. We find a partial fraction expansion for the ratio of polynomials in $s$ on the right side of the last expression above:

$$\frac{s+8}{(s+10)(s+40)} = \frac{A}{s+10} + \frac{B}{s+40}$$

Using the pole cover-up method, we compute $A$ and $B$:

$$A = (s+10)\left.\frac{s+8}{(s+10)(s+40)}\right|_{s=-10} = \frac{-10+8}{(-10+40)} = -\frac{1}{15}$$

$$B = (s+40)\left.\frac{s+8}{(s+10)(s+40)}\right|_{s=-40} = \frac{-40+8}{(-40+10)} = \frac{16}{15}$$

Substituting into $V_o(s)$, we have the following partial fraction version:

$$V_o(s) = -5V\left(\frac{-1}{s+10} + \frac{16}{s+40}\right)$$

Taking the inverse Laplace transform yields our final answer:

$$v_o(t \geq 0) = 15e^{-10t} - 80e^{-40t} u(t) \text{ V}$$

**NOTE:** We could omit the $u(t)$, but it reminds us that our answer only applies to $t \geq 0$. 