HOMEWORK #13 prob 1b solution

EX: Find $\mathcal{L}\{\delta(t-4)u(t-4)+t\cos(9t)\}$.

SOL'N: a) We use the delay transform for the first part of the expression and the identity for multiplication by t for the second part of the expression:

The delay identity is as follows:

$$\mathcal{L}\{f(t-a)u(t-a)\}=e^{-as}\mathcal{L}\{f(t)\}$$

Here, we have the following result:

$$\mathcal{L}\{\delta(t-4)u(t-4)\}=e^{-4s}\mathcal{L}\{\delta(t)\}=e^{-4s}$$

NOTE: The multiplication by the delayed step function actually has no effect on the delayed delta function.

Next, we apply the following identity for multiplication by t:

$$\mathcal{L}\{tf(t)\}=-\frac{d}{ds}F(s)$$

Here, we have the following result:

$$\mathcal{L}\{t\cos(9t)\}=-\frac{d}{ds}\frac{s}{s^2+9^2}=\frac{-1}{s^2+9^2}+\frac{s(2s)}{[s^2+9^2]^2}$$

We sum the results for the final answer:

$$\mathcal{L}\{\delta(t-4)u(t-4)+t\cos(9t)\}=e^{-4s}-\frac{1}{s^2+9^2}+\frac{2s^2}{[s^2+9^2]^2}$$