HOMEWORK #13 prob 1a solution

Ex: Find $\mathcal{L}\left\{\left(\int_0^{t-2} \tau e^{-3\tau}\, d\tau\right)u(t-2)\right\}$.

SOL'N:

We identify $\int_0^{t-2} \tau e^{-3\tau}\, d\tau$ as $f(t-2)$ and use the delay (or shift) identity:

$$\mathcal{L}\{f(t-2)u(t-2)\} = e^{-2s} F(s)$$

To find $f(t)$, we replace $t-2$ with t in $f(t-2)$:

$$f(t) = \int_0^{t} \tau e^{-3\tau}\, d\tau$$

To find $\mathcal{L}\{\int_0^t \tau e^{-3\tau}\, d\tau\}$, we use the integral identity:

$$\mathcal{L}\{\int_0^t f(t)\, dt\} = \frac{1}{s} F(s)$$

where $F(s) = \mathcal{L}\{f(t)\} = \mathcal{L}\{\tau e^{-3\tau}\}$.

From tables, we have $\mathcal{L}\{\tau e^{-3\tau}\} = \dfrac{1}{(s+3)^2}$.

Back substituting into earlier expressions gives

$$\mathcal{L}\left\{\left(\int_0^{t-2} \tau e^{-3\tau}\, d\tau\right)u(t-2)\right\} = \frac{e^{-2s}}{s(s+3)^2}$$