EX: Find the inverse Laplace transform for the following expression:

$$F(s) = \frac{-3s^2 + 99s - 1200}{s^3 + 11s^2 + 100s + 1100}$$

SOL'N: First, we need to factor the denominator. This could be done by using a computer or by using the formula for a cubic equation. We find that 11 is one factor, after which we have the following form:

$$F(s) = \frac{-3s^2 + 99s - 1200}{(s+11)(s^2+100)}$$

Now we use partial fractions, but we write the complex root terms as a pure cosine plus a pure sine. The frequency of the cosine and sine will be $\sqrt{100} = 10$.

$$F(s) = \frac{A}{s+11} + \frac{Bs + C(10)}{s^2 + 100}$$

Putting terms over a common denominator gives

$$F(s) = \frac{A(s^2+100) + (Bs+10C)(s+11)}{(s+11)(s^2+100)}$$

Now we match the numerator of $F(s)$ with the original expression for $F(s)$.

$$A(s^2+100) + (Bs+10C)(s+11) = -3s^2 + 99s - 1200$$

or

$$As^2 + 100A + Bs^2 + 11Bs + 10Cs + 110C = -3s^2 + 99s - 1200$$

or

$$(A+B)s^2 + (11B+10C)s + 100A + 110C = -3s^2 + 99s - 1200$$

The coefficients for each power of s must match.

$$A + B = -3$$

$$11B + 10C = 99$$

$$100A + 110C = 1200$$

This will be tedious to solve, so we find A by the pole cover-up method.

$$A = F(s)(s+11)\big|_{s=-11} = \left.\frac{-3s^2 + 99s - 1200}{s^2 + 100}\right|_{s=-11}$$

$$= \frac{-3(121) - 99(11) - 1200}{121 + 100} = \frac{-2652}{221} = -12$$

Now we easily find *B* and *C*.

$$B = 9$$

$$C = 0$$

So we have

$$F(s) = -12\frac{1}{s+11} + 9\frac{s}{s^2 + 100}$$

and

$$f(t) = \left[12e^{-11t} + 9\cos(10t)\right]u(t).$$