HOMEWORK #12 prob 3b solution

EX: Find the inverse Laplace transform for the following expression:

$$F(s) = \frac{7s + 70}{s^2 + 8s + 25}$$

SOL'N: First we write the function with a denominator that is effectively factored to show roots $s + 4 \pm j3$.

$$F(s) = \frac{7s + 70}{s^2 + 8s + 25} = \frac{7s + 70}{(s+4)^2 + 3^2}$$

Now we match this function to a decaying cosine and sine:

$$F(s) = \frac{7s + 70}{(s+4)^2 + 3^2} = A\frac{s+4}{(s+4)^2 + 3^2} + B\frac{3}{(s+4)^2 + 3^2}$$

or

$$F(s) = \frac{A(s+4) + B(3)}{(s+4)^2 + 3^2}$$

We match coefficients of powers of s in the numerator.

$$As = 7s \quad \text{and} \quad A(4) + B(3) = 70$$

or

$$A = 7 \quad \text{and} \quad 7(4) + B(3) = 70 \quad \text{which gives} \quad B = 14$$

So

$$F(s) == 7\frac{s+4}{(s+4)^2 + 3^2} + 14\frac{3}{(s+4)^2 + 3^2}$$

and

$$f(t) = \left[7e^{-4t}\cos(3t) + 14e^{-4t}\sin(3t)\right]u(t)$$