HOMEWORK #12 prob 3a solution

EX: Find the inverse Laplace transform for the following expression:

$$F(s)=\frac{5s+18}{s^2+6s}$$

SOL'N: We use partial fractions. We factor the denominator to get the roots for the partial fraction terms.

$$F(s)=\frac{5s+18}{s^2+6s}=\frac{5s+18}{s(s+6)}=\frac{A}{s}+\frac{B}{s+6}$$

We use the pole cover-up method to find *A* and *B*:

$$A=F(s)s\Big|_{s=0}=\frac{5s+18}{s+6}\Bigg|_{s=0}=\frac{18}{6}=3$$

$$B=F(s)(s+6)\Big|_{s=-6}=\frac{5s+18}{s}\Bigg|_{s=-6}=\frac{-30+18}{-6}=2$$

Thus, we have

$$F(s)=\frac{3}{s}+\frac{2}{s+6}$$

Taking the inverse transform, we have our answer:

$$f(t)=\mathcal{L}^{-1}\left\{\frac{3}{s}+\frac{2}{s+6}\right\}=(3+2e^{-6t})u(t)$$

NOTE: By convention, we multiply our answer by $u(t)$ to remind ourselves that we cannot know what the value of $f(t)$ is before time zero, since the Laplace transform only takes into account values for $t>0^-$. The actual value of $f(t)$ is unknown, so we set it to zero.

NOTE: $u(t)\cdot u(t)=u(t)$