HOMEWORK #12 prob 1d solution

EX: Find the Laplace transforms of the following waveform:

$$t\int_0^t te^{-at}\,dt$$

SOL'N: We work from the inside out. We start with the Laplace transform of te^{-at}, which is found in tables.

$$\mathcal{L}\{te^{-at}\} = \frac{1}{(s+a)^2}$$

Next, we apply the integral identity:

$$\mathcal{L}\left\{\int_0^t v(t)\,dt\right\} = \frac{V(s)}{s}$$

Here, we have the following result:

$$\mathcal{L}\left\{\int_0^t te^{-at}\,dt\right\} = \frac{1}{s(s+a)^2}$$

Finally, we apply the identity for multiplication by t:

$$\mathcal{L}\{tv(t)\} = -\frac{dV(s)}{ds}$$

Here, applying this identity yields our final answer:

$$\mathcal{L}\left\{t\int_0^t te^{-at}\,dt\right\} = -\frac{d}{ds}\frac{1}{s(s+a)^2} = -\frac{d}{ds}\left[s^{-1}(s+a)^{-2}\right]$$

$$= -[(-1)s^{-2}(s+a)^{-2} + s^{-1}(-2)(s+a)^{-3}]$$

$$= \frac{1}{s^2(s+a)^2} + \frac{2}{s(s+a)^3}$$

$$= \frac{s+a}{s^2(s+a)^3} + \frac{2s}{s^2(s+a)^3}$$

$$= \frac{3s+a}{s^2(s+a)^3}$$