Ex: Find the Laplace transforms of the following waveform:

\[ t \int_0^t e^{-at} \, dt \]

**Sol’N:** We work from the inside out. We start with the Laplace transform of \( te^{-at} \), which is found in tables.

\[ \mathcal{L}\{te^{-at}\} = \frac{1}{(s + a)^2} \]

Next, we apply the integral identity:

\[ \mathcal{L}\left[ \int_0^t v(t) \, dt \right] = \frac{V(s)}{s} \]

Here, we have the following result:

\[ \mathcal{L}\left[ t \int_0^t e^{-at} \, dt \right] = \frac{1}{s(s + a)^2} \]

Finally, we apply the identity for multiplication by \( t \):

\[ \mathcal{L}\{tv(t)\} = -\frac{dV(s)}{ds} \]

Here, applying this identity yields our final answer:

\[ \mathcal{L}\left[ t \int_0^t e^{-at} \, dt \right] = \frac{1}{ds} \left[ \frac{1}{s(s + a)^2} \right] = \frac{d}{ds} \left[ \frac{1}{s(s + a)^2} \right] \]

\[ = \frac{1}{s^2(s + a)^2} + \frac{2}{s(s + a)^3} \]

\[ = \frac{s + a}{s^2(s + a)^3} + \frac{2s}{s^2(s + a)^3} \]

\[ = \frac{3s + a}{s^2(s + a)^3} \]