EX: Find the Laplace transform of the following waveform:

$$f(t) = t\cos(\omega t)e^{-at}$$

SOL'N: We start with the Laplace transform of the cosine and then apply the identity for multiplication by t and then the identity for multiplication by e^{-at}.

$$\mathcal{L}\{\cos(\omega t)\} = \frac{s}{s^2+\omega^2}$$

Now we use the identity for multiplication by t:

$$\mathcal{L}\{tv(t)\} = -\frac{dV(s)}{ds}$$

Here, this gives the following result:

$$\mathcal{L}\{t\cos(\omega t)\} = -\frac{d}{ds}\frac{s}{s^2+\omega^2} = -\frac{1}{s^2+\omega^2} + \frac{s2s}{(s^2+\omega^2)^2}$$
$$= \frac{-(s^2+\omega^2)+2s^2}{(s^2+\omega^2)^2} = \frac{s^2-\omega^2}{(s^2+\omega^2)^2}$$

Now we apply the identity for multiplication by e^{-at}:

$$\mathcal{L}\{v(t)e^{-at}\} = V(s+a)$$

Here, this gives the following result:

$$\mathcal{L}\left\{t\cos(\omega t)e^{-at}\right\} = \frac{(s+a)^2-\omega^2}{[(s+a)^2+\omega^2]^2}$$