

EX: Compute the Laplace transform of the following functions by calculating the integral expression for the Laplace transform (step-by-step by hand):

a)  $f(t) = u(t) - u(t-1)$  where  $u(t)$  is the unit step function:  $u(t) = \begin{cases} 0 & t < 0 \\ 1 & 0 \leq t \end{cases}$

b)  $f(t) = \begin{cases} 0 & t < 0 \\ e^{-2t} & 0 \leq t \end{cases}$

c)  $f(t) = \begin{cases} 0 & t < 4 \\ e^{-2(t-4)} & 4 \leq t \end{cases}$

SOL'N: a) We can separate the integral into two pieces.

$$F(s) = \int_{0^-}^{\infty} [u(t) - u(t-1)]e^{-st} dt = \int_{0^-}^{\infty} u(t)e^{-st} dt - \int_{0^-}^{\infty} u(t-1)e^{-st} dt$$

The step function is equal to unity or zero and has the effect of changing the limits on the integral.

$$F(s) = \int_{0^-}^{\infty} e^{-st} dt - \int_1^{\infty} e^{-st} dt = \frac{e^{-st}}{-s} \Big|_{0^-}^{\infty} - \frac{e^{-st}}{-s} \Big|_1^{\infty}$$

We assume that the value of  $s$  has a positive real part so that the upper limit of the integral equals zero.

$$F(s) = \frac{0}{-s} - \frac{e^{-s0^-}}{-s} - \left[ \frac{0}{-s} - \frac{e^{-s1}}{-s} \right] = \frac{1}{s} - \frac{e^{-1}}{s} = \frac{1 - e^{-1}}{s}$$

b)

$$F(s) = \int_{0^-}^{\infty} e^{-2t} e^{-st} dt = \frac{e^{-(2+s)t}}{-(2+s)} \Big|_{0^-}^{\infty} = \frac{0}{-(2+s)} - \frac{e^0}{-(2+s)}$$

We assume that the value of  $s$  has a positive real part so that the upper limit of the integral equals zero.

$$F(s) = \frac{1}{s+2}$$

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c) We set the lower limit of the integral to zero to account for the part of  $f(t)$  that is zero.

$$F(s) = \int_4^{\infty} e^{-2(t-4)} e^{-st} dt = \int_4^{\infty} e^{-2t} e^8 e^{-st} dt = e^8 \int_4^{\infty} e^{-2t} e^{-st} dt$$

As before, we assume that the value of  $s$  has a positive real part so that the upper limit of the integral equals zero.

$$F(s) = e^8 \left. \frac{e^{-(2+s)t}}{-(2+s)} \right|_4^{\infty} = e^8 \left[ \frac{0}{-(2+s)} - \frac{e^{-(2+s)4}}{-(2+s)} \right] = \frac{e^{-4s}}{s+2}$$