ECE 2240 **HOMEWORK #10 prob 2 solution**
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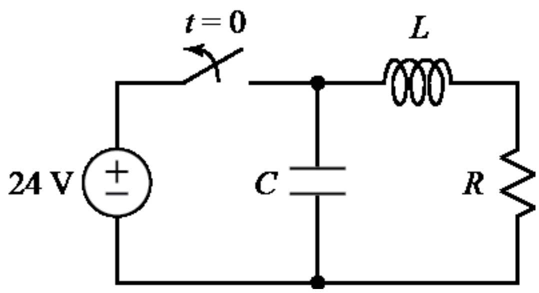
Ex:



$C = 1$ mF $L = 25$ mH

A relay is driven by a 24 V power supply, as shown above. Power is turned off at $t = 0$. The current, $i(t)$, for $t > 0$ has two terms that decay exponentially without oscillation. One term dies out quickly, and the other term dies out with a time constant of $\tau = 10$ ms, as in $e^{-t/10\text{ms}}$. Given the time constant and the information in the diagram above, find the value of R.

sol'n: From what we are told, we have one characteristic root equal to $-1/10$ms.

Since the circuit is a series RLC, we use $\alpha = R/(2L)$.

As always, $\omega_0^2 = 1/(LC)$.

Our characteristic roots are

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

The problem states that the term for the root equal to $-1/10$ms dies out more slowly. This means the magnitude of this root is smaller. This is, therefore, the root corresponding to

$$s = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

So we have $s = -\alpha + \sqrt{\alpha^2 + \omega_0^2} = -1/10\text{ms}$.

We solve for R.

$$-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} = -\frac{1}{10\,ms} = -100$$

We get the $\sqrt{\;}$ by itself on one side by adding $\frac{R}{2L}$ to both sides.

$$\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} = -100 + \frac{R}{2L}$$

We now square both sides.

$$\cancel{\left(\frac{R}{2L}\right)^2} - \frac{1}{LC} = +100^2 - 2(100)\frac{R}{2L} + \cancel{\left(\frac{R}{2L}\right)^2}$$

where $\frac{1}{LC} = \frac{1}{25\,m\;1\,m} = \left(\frac{1}{5m}\right)^2 \,(r/s)^2$

or

$$\frac{1}{LC} = 200^2 \;r^2/s^2$$

We rearrange to get R by itself on one side.

$$200\,\frac{R}{2L} = 100^2 + 200^2 = 50k\;r^2/s^2$$

$$R = 50k \cdot \frac{2L}{200} \quad \Omega/H$$

$$R = \frac{50k}{200} \cdot 2(25m) = \frac{25}{2}\;\Omega$$

or

$$R = 12.5\;\Omega$$