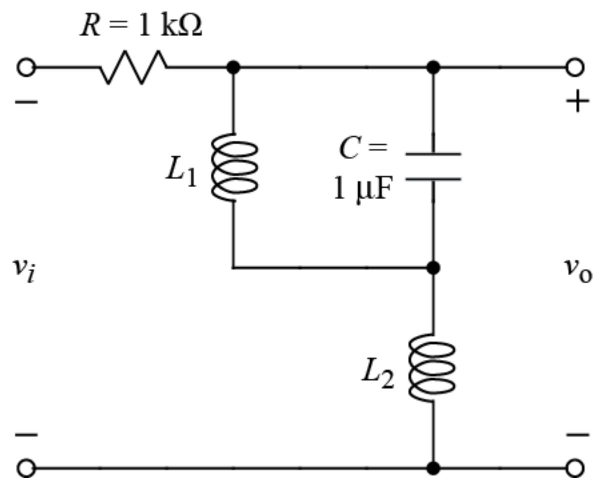
Ex:



The above filter circuit is intended to be complimentary to the filter designed in ECE 2240 Lab 4. That is, it is designed to have low gain at frequencies where the filter in Lab 4 had high gain, and vice versa.

a) Find values of $L_1 \neq 0$ and $L_2 \neq 0$ such that the magnitude of the filter's transfer function, H, has the following values:

$$|H(j\omega)| = 1 \text{ at } \omega = 2\pi \cdot 1 \text{ kHz}$$

$$|H(j\omega)| = 0 \text{ at } \omega = 2\pi \cdot 3 \text{ kHz}$$

b) Using the values of L_1 and L_2 you found in part (a), find the approximate magnitude of $H(j\omega)$ at $\omega = 2\pi \cdot 5$ kr/s. Your answer should be within 15% of the actual value.

bonus) Describe how increasing the value of R would affect the shape of the plot of the gain, $|H(j\omega)|$, versus ω.

SOL'N: a) The parallel combination of L_1 and C, becomes an open circuit at resonant frequency, which is here called ω_1.

$$\frac{1}{\dfrac{1}{j\omega_1 L_1} + j\omega_1 C} = \infty$$

The solution is for the denominator to be zero, which is the same as the resonant frequency:

$$\omega_1 = \frac{1}{\sqrt{L_1 C}}$$

or

$$\omega_1^2 = \frac{1}{L_1 C}$$

When L_1 and C open up, our gain will be unity, since no current will flow through the vertical impedance and there will be no voltage drop across R_1. So we want ω_1 to be 1 kHz.

$$L_1 = \frac{1}{\omega_1^2 C} = \frac{1}{(2\pi)^2 (1\text{k})^2 1\mu}\text{H} = \frac{1}{(2\pi)^2}\text{H} \cong 25.3\,\text{mH}$$

For the second constraint, we want the gain to be zero, meaning that the entire vertical impedance should short out.

$$\frac{1}{\dfrac{1}{j\omega_2 L_1} + j\omega_2 C} + j\omega_2 L_2 = 0$$

We solve for L_2, using $\omega_2 = 2\pi{\cdot}3\text{k}$ r/s and the value of L_1 from above.

$$L_2 = -\frac{1}{j\omega_2}\frac{1}{\dfrac{1}{j\omega_2 L_1} + j\omega_2 C} = -\frac{1}{\dfrac{1}{L_1} - \omega_2^2 C} = \frac{1}{(2\pi)^2 (3\text{k})^2 1\mu - \dfrac{1}{25\text{m}}}$$

or

$$L_2 \approx \frac{1}{(2\pi)^2 (9) - 40} \approx 3.17\ \text{mH} \approx 3.2\ \text{mH}$$

b) This is merely a calculation involving complex numbers. First, we simplify the transfer function expression.

$$H(j\omega) = \frac{j\omega L_2 + j\omega L_1 \parallel \dfrac{1}{j\omega C}}{j\omega L_2 + j\omega L_1 \parallel \dfrac{1}{j\omega C} + R} = \frac{1}{1 + \dfrac{R}{j\omega L_2 + j\omega L_1 \parallel \dfrac{1}{j\omega C}}}$$

or

$$H(j\omega) = \cfrac{1}{1 + \cfrac{R}{j\omega L_2 + \cfrac{1}{\cfrac{1}{j\omega L_1} + j\omega C}}}$$

where

$$j\omega C = j(2\pi)5\text{k}(1\mu)\Omega \approx j10\pi\,\text{m}\Omega$$

$$j\omega L_1 = j(2\pi)5\text{k}(25\text{m})\Omega \approx j250\pi\,\Omega$$

$$j\omega L_2 = j(2\pi)5\text{k}(3.2\text{m})\Omega \approx j32\pi\,\Omega$$

So we have the following calculation.

$$H(j2\pi\cdot 5\text{kr/s}) \approx \cfrac{1}{1 + \cfrac{1\text{k}}{j32\pi + \cfrac{1}{\cfrac{1}{j250\pi} + j10\text{m}\pi}}}$$

or

$$H(j2\pi\cdot 5\text{kr/s}) \approx \cfrac{1}{1 + \cfrac{1\text{k}}{j32\pi + \cfrac{1}{-\cfrac{j4\text{m}}{\pi} + j10\text{m}\pi}}} \approx \cfrac{1}{1 + \cfrac{1\text{k}}{j32\pi + \cfrac{1}{j10\text{m}\pi}}}$$

or

$$H(j2\pi\cdot 5\text{kr/s}) \approx \cfrac{1}{1 + \cfrac{1\text{k}}{j32\pi - j\cfrac{100}{\pi}}} \approx \cfrac{1}{1 + \cfrac{1\text{k}}{j32\pi - j32}}$$

or

$$H(j2\pi\cdot 5\text{kr/s}) \approx \cfrac{1}{1 + \cfrac{1\text{k}}{j32\pi - j32}} \approx \cfrac{1}{1 + \cfrac{1\text{k}}{j66.7}} \approx \frac{1}{1 - j15} \approx j\frac{1}{15}$$

or

$$\left|H(j2\pi\cdot 5\text{kr/s})\right| \approx \frac{1}{15}$$

bonus) In general, increasing R reduces v_o and reduces the gain. The maximum gain at 1 kHz and the minimum gain at 3 kHz still exist with the same values as before. This means that the peak at 1 kHz will be narrower and the dip at 3 kHz will be broader.