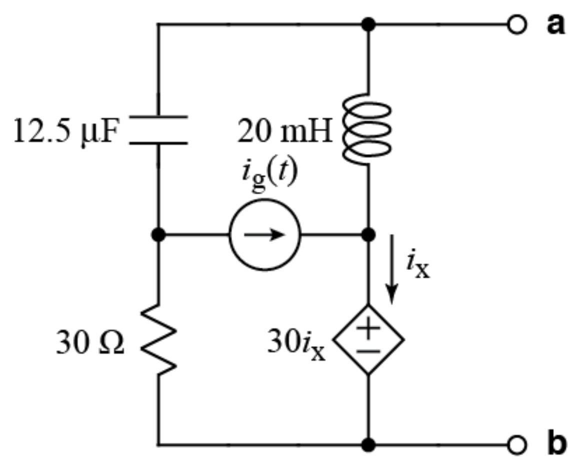
EXAM #4 prob 1 solution

Ex:



$$i_g(t) = 20\cos(2\text{k}t) \text{ A}$$

a) Draw a frequency-domain equivalent of the above circuit. Show a numerical phasor value for $i_g(t)$, and show numerical impedance values for R, L, and C. Label the dependent source appropriately.

b) Find the Thevenin equivalent (in the frequency domain) for the above circuit relative to terminals **a** and **b**. Give the numerical phasor value for $\mathbf{V}_{Th}$ and the numerical impedance value of z_{Th}.

SOL'N: a) The phasor for the current source is a real number (no phase shift).

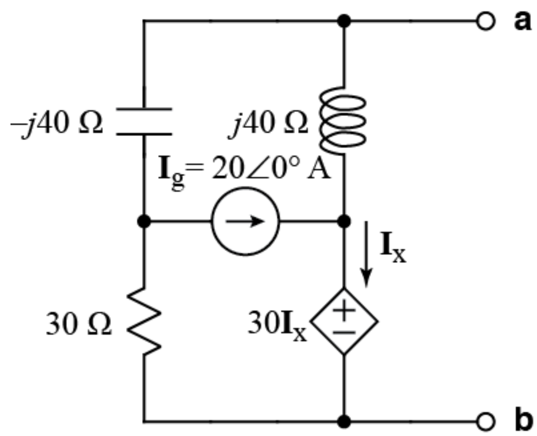
$$\mathbf{I}_g = 20\angle 0° \text{ A}$$

From the expression for $i_g(t)$, we see that ω = 2 kr/s. We use this to calculate the impedance of the L and C.

$$j\omega L = j2\text{k}(20\text{m})\ \Omega = j40\ \Omega$$

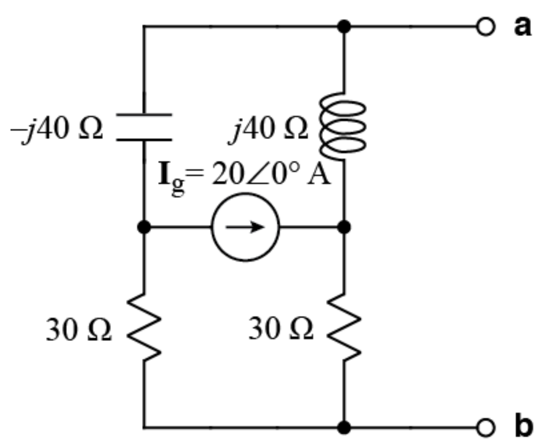
$$\frac{1}{j\omega C} = \frac{1}{j2\text{k}(12.5\mu)}\ \Omega = \frac{1}{j25\text{m}} = -j40\ \Omega$$

The dependent source just outputs 30 times the phasor $\mathbf{I}_x$.



b) We may replace the dependent source with a resistance since we have both the voltage and the current in terms of the current.

$$z_{eq} = \frac{30\mathbf{I}_x}{\mathbf{I}_x} = 30\ \Omega$$



The impedance in the top half of the circuit is 0 Ω. Consequently, all the current from the current source will flow in the top half of the circuit, and no current will flow around the bottom half.

The Thevenin equivalent voltage is z_{ab}. The voltage drop from **a** to **b** will be the voltage drop across the bottom right 30 Ω resistor, which is zero amps times 30 Ω, plus the voltage drop across the top right $j40$ Ω impedance, which is $-\mathbf{I}_x(j40\ \Omega)$.

$$\mathbf{V}_{Th} = \mathbf{V}_{ab} = -20\angle 0°\,\text{A}\,(j40\,\Omega) = -j800\,\text{V}$$

To find the Thevenin impedance, we turn off the current source, which becomes an open circuit, and look in from the **a** and **b** terminals. We see the left and right branches in parallel.

$$z_{\text{Th}} = (30 - j40\,\Omega) \parallel (30 + j40\,\Omega)$$

or

$$z_{\text{Th}} = \frac{1}{\dfrac{1}{30 - j40\,\Omega} + \dfrac{1}{30 + j40\,\Omega}}$$

or

$$z_{\text{Th}} = \frac{1}{\dfrac{30 + j40}{30^2 + 40^2} + \dfrac{30 - j40}{30^2 + 40^2}}\ \Omega$$

or

$$z_{\text{Th}} = \frac{1}{\dfrac{30 + j40}{50^2} + \dfrac{30 - j40}{50^2}}\ \Omega$$

or

$$z_{\text{Th}} = \frac{50^2}{30 + j40 + 30 - j40}\ \Omega$$

or

$$z_{\text{Th}} = \frac{2500}{60}\ \Omega = \frac{125}{3}\ \Omega \approx 41.67\,\Omega$$