

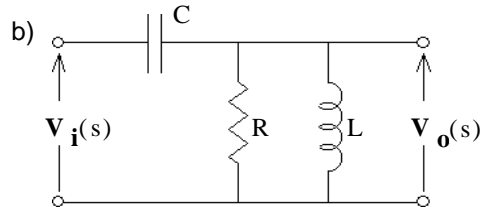
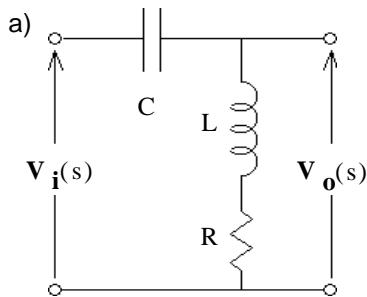
ECE 2210 homework # 15

Due:

b

1. Read the *Second-Order Transients* handout.

2. Find the transfer function $\mathbf{H}(s) = \frac{\mathbf{V}_o(s)}{\mathbf{V}_i(s)}$ for these circuits. Write $\mathbf{H}(s)$ in the normal form, as shown below.



$$\mathbf{H}(s) = K \cdot \frac{s^n + k_1 \cdot s^{n-1} + \dots + k_{n-1}}{s^m + c_1 \cdot s^{m-1} + \dots + c_{m-1}}$$

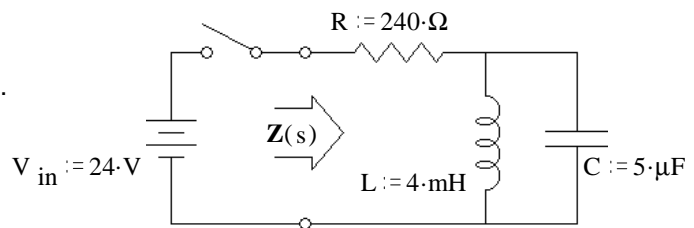
3. Write the characteristic equation for each of the circuits in problem 2.

4. For the circuit shown, with a disconnected source:

a) Find the generalized impedance of the circuit, $\mathbf{Z}(s)$.

b) What is the impedance at $s = 0$?

c) What is the impedance at $s = \infty$?



d) When the switch is closed, current will begin to flow. The voltage source is the input and the current (through R) can be considered the "output" (i.e. caused by the input). Find the transfer function and the natural frequencies of the circuit (s solutions to the characteristic equation).

e) What is the character of the response (undamped, underdamped, critically damped, or overdamped)?

f) Find the initial values of all three currents.

g) Find the final values of all three currents

ECE 2210 homework # 16

Due:

1. A series RLC circuit with $R = 200 \Omega$, $L = 0.1 \text{ H}$ and $C = 100 \mu\text{F}$ has a constant voltage $V = 20$ volts applied at $t = 0$. The capacitor has no initial charge.

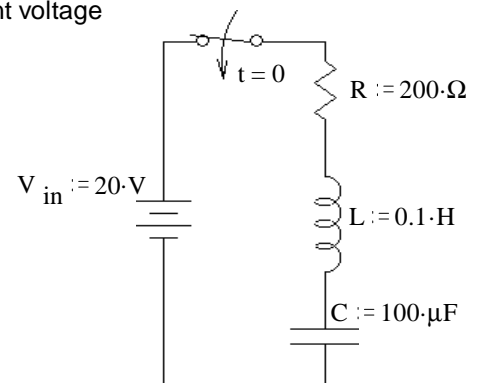
a) Find the characteristic equation of the circuit at right. (hint: take $i(t)$ as the "output")

b) Find the solutions to the characteristic equation.

c) Is this circuit over, under, or critically damped?

d) The switch is switched down at time $t = 0$. Find the final and initial conditions: final: $i(\infty)$ initial: $i(0)$, $v_C(0)$ and $\frac{d}{dt}i(0)$

e) Write the full expression for $i(t)$, including all the constants that you find.



2. A series RLC circuit with $R = 200 \Omega$, $L = 0.1 \text{ H}$ and $C = ? \mu\text{F}$ is to be made critically damped by the selection of the capacitance. Find the required value of C .

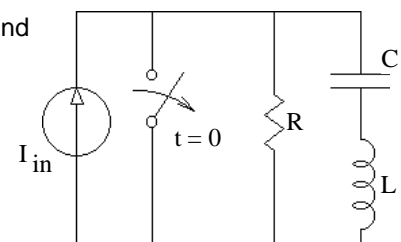
3. Find the natural frequency of a series RLC circuit in which $R = 200 \Omega$, $L = 0.1 \text{ H}$ and $C = 5 \mu\text{F}$. (The natural frequency is the ω part of $s_1 = \alpha + j\omega$)

4. In the circuit shown, when the switch is opened, the current I_{in} (current source) is forced to flow through the circuit.

a) Write a differential equation for i_L . Hint: use LaPlace impedance method.

b) Write a differential equation for v_C .

c) Find the characteristic equation for the circuit shown.



1. For the circuit at right:

- a) Find the characteristic equation of the circuit at right.
- b) Find the solutions to the characteristic equation.
- c) Is this circuit over, under, or critically damped?
- d) The switch has been in the top position for a long time and is switched down at time $t = 0$. Find the final and initial conditions:

$$v_C(\infty), i_L(\infty), v_C(0), i_L(0), \frac{d}{dt}v_C(0) \text{ and } \frac{d}{dt}i_L(0)$$

e) Write the full expression for $i_L(t)$, including all the constants that you find.

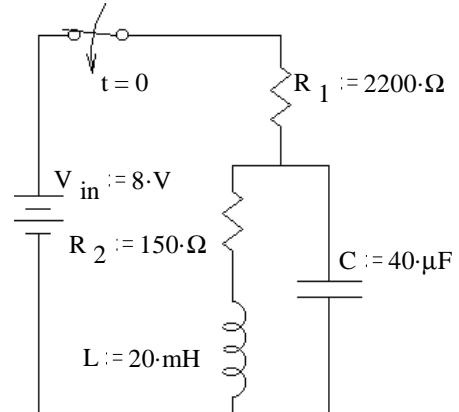
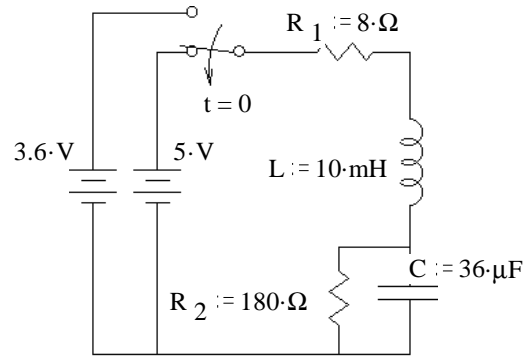
2. a) Find the characteristic equation of the circuit at right.

- b) Find the solutions to the characteristic equation.
- c) Is this circuit over, under, or critically damped?
- d) Find the final conditions: $i_L(\infty)$ and $v_C(\infty)$

e) The switch has been open for a long time and is switched down at time $t = 0$. Find the initial conditions:

$$i_L(0), v_C(0) \text{ and } \frac{d}{dt}v_C(0)$$

f) Write the full expression for $v_C(t)$, including all the constants.



Answers

HW 15
2.a) $H(s) = \frac{s^2 + \frac{R}{L} \cdot s}{s^2 + \frac{R}{L} \cdot s + \frac{1}{L \cdot C}}$

b) $H(s) = \frac{s^2}{s^2 + \frac{1}{C \cdot R} \cdot s + \frac{1}{L \cdot C}}$

3.a) $0 = s^2 + \frac{R}{L} \cdot s + \frac{1}{L \cdot C}$ b) $0 = s^2 + \frac{1}{C \cdot R} \cdot s + \frac{1}{L \cdot C}$

4.a) $\frac{240 \cdot \Omega \cdot s^2 + 2 \cdot 10^5 \cdot \frac{\Omega}{\text{sec}} \cdot s + 1.2 \cdot 10^{10} \cdot \frac{\Omega}{\text{sec}^2}}{s^2 + 5 \cdot 10^7 \cdot \frac{1}{\text{sec}^2}}$

b) 240·Ω

c) 240·Ω

d) $\frac{\frac{1}{240 \cdot \Omega} \cdot s^2 + 2.083 \cdot 10^5 \cdot \frac{1}{\Omega \cdot \text{sec}^2}}{s^2 + 833.333 \cdot \frac{1}{\text{sec}} \cdot s + 5 \cdot 10^7 \cdot \frac{1}{\text{sec}^2}}$

s values below

$(-417 + 7059j) \cdot \frac{1}{\text{sec}}$ $(-417 - 7059j) \cdot \frac{1}{\text{sec}}$

e) underdamped

f) R 0.1·A g) R 0.1·A
L 0·A L 0.1·A
C 0.1·A C 0·A

HW 16

1. a) $0 = s^2 + \frac{R}{L} \cdot s + \frac{1}{L \cdot C}$ b) $-51.3 \cdot \frac{1}{\text{sec}}$ $-1949 \cdot \frac{1}{\text{sec}}$ c) overdamped

d) $i(\infty) = 0 \cdot \text{A}$ $i(0) = 0$ $v_C(0) = 0$ $\frac{d}{dt}i(0) = 200 \cdot \frac{\text{A}}{\text{sec}}$ e) $i(t) = 0.1054 \cdot e^{-\frac{51.3}{\text{sec}} \cdot t} - 0.1054 \cdot e^{-\frac{1949}{\text{sec}} \cdot t}$

2. 10·μF 3. 159·Hz

4. a) $\frac{R}{L} \cdot \frac{d}{dt}i_{in} = \frac{d^2}{dt^2}i_L + \frac{R}{L} \cdot \frac{d}{dt}i_L + \frac{1}{L \cdot C} \cdot i_L$ b) $\frac{R}{L \cdot C} \cdot i_{in} = \frac{d^2}{dt^2}v_C + \frac{R}{L} \cdot \frac{d}{dt}v_C + \frac{1}{L \cdot C} \cdot v_C$ c) $s^2 + \frac{R}{L} \cdot s + \frac{1}{L \cdot C} = 0$

HW 17

1. a) $s^2 + \left(\frac{R_1}{L} + \frac{1}{C \cdot R_2}\right) \cdot s + \left(\frac{1}{C \cdot L} + \frac{R_1}{R_2 \cdot C \cdot L}\right) = 0$ b) $s_1 := (-477 + 1635j) \cdot \frac{1}{\text{sec}}$ $s_2 := (-477 - 1635j) \cdot \frac{1}{\text{sec}}$

c) underdamped d) 4.79·V 26.6·mA 3.45·V 19.15·mA $0 \cdot \frac{\text{V}}{\text{sec}}$ $140 \cdot \frac{\text{A}}{\text{sec}}$

e) $i_L(t) = 26.6 \cdot \text{mA} + e^{-\frac{477.2}{\text{sec}} \cdot t} \cdot \left(-7.45 \cdot \cos\left(\frac{1635}{\text{sec}} \cdot t\right) + 83.45 \cdot \sin\left(\frac{1635}{\text{sec}} \cdot t\right)\right) \cdot \text{mA}$

2a) $0 = s^2 + \left(\frac{R_2}{L} + \frac{1}{R_1 \cdot C}\right) \cdot s + \left(\frac{1}{L \cdot C} + \frac{R_2}{R_1 \cdot L \cdot C}\right)$ b) $s_1 := -182.2 \cdot \frac{1}{\text{sec}}$ $s_2 := -7329 \cdot \frac{1}{\text{sec}}$ c) overdamped

d) $i_L(\infty) = 3.404 \cdot \text{mA}$ $v_C(\infty) = 0.511 \cdot \text{V}$ e) $i_L(0) = 0 \cdot \text{mA}$ $v_C(0) = 0 \cdot \text{V}$ $\frac{d}{dt}v_C(0) = 90.91 \cdot \frac{\text{V}}{\text{sec}}$

f) $v_C(t) = 0.511 \cdot \text{V} - 0.511 \cdot \text{V} \cdot e^{-182.2 \cdot t} + 0.000295 \cdot \text{V} \cdot e^{-7329 \cdot t}$ **ECE 2210 homework # 15 through 17**