Notes, Second Order Transients

ECE 2210 A.Stolp 4/6/00, 3/16/07

			Laplace impedance	initial conditions	final conditions DC	final conditions AC
Capacitor	$i_C = C\cdot\frac{d}{dt}v_C$	$v_C = \frac{1}{C}\cdot\int_{-\infty}^{t} i_C\,dt$	$\frac{1}{C\cdot s}$	short, or $v_C(0+) = v_C(0-)$	open	$\frac{1}{j\cdot\omega\cdot C}$
Inductor	$v_L = L\cdot\frac{d}{dt}i_L$	$i_L = \frac{1}{L}\cdot\int_{-\infty}^{t} v_L\,dt$	$L\cdot s$	open, or $i_L(0+) = i_L(0-)$	short	$j\cdot\omega\cdot L$
Resistor	$v_R = i_R\cdot R$	$i_R = \frac{v_R}{R}$	R	R	R	R

Characteristic equation

Use Laplace impedances, manipulate your circuit equation(s) into one equation of this form:

May be $\mathbf{I_{in}}$ or any forcing function — May be $\mathbf{I_X}$ or any desired variable

$$\left(a_1\cdot s^2 + b_1\cdot s + k_1\right)\cdot \mathbf{V_{in}}(s) = \left(s^2 + b\cdot s + k\right)\cdot \mathbf{V_X}(s)$$ with NO denominator s terms

a_1, b_1, k_1 coefficients may be zero

$s^2 + b\cdot s + k = 0$ is the characteristic equation

Differential equation

$$a_1\cdot\frac{d^2}{dt^2}v_{in}(t) + b_1\cdot\frac{d}{dt}v_{in}(t) + k_1\cdot V_{in}(t) = \frac{d^2}{dt^2}v_X(t) + b\cdot\frac{d}{dt}v_X(t) + k\cdot v_X(t)$$

Transfer function

Rearrange circuit equation to: $\mathbf{H}(s) = \frac{\text{output}}{\text{input}} = \frac{\mathbf{V_X}(s)}{\mathbf{V_{in}}(s)} = \frac{a_1\cdot s^2 + b_1\cdot s + k_1}{s^2 + b\cdot s + k}$ = transfer function

(May be $\mathbf{I_X}$ or any desired variable; May be $\mathbf{I_{in}}$ or any forcing function)

$s^2 + b\cdot s + k = 0$ characteristic equation

Complete solution

Solutions to the characteristic equation: $s_1 = -\frac{b}{2} + \frac{\sqrt{b^2 - 4\cdot k}}{2}$ $\qquad s_2 = -\frac{b}{2} - \frac{\sqrt{b^2 - 4\cdot k}}{2}$

Find initial Conditions (v_C and/or i_L)

Find conditions of just before time $t = 0$, $v_C(0-)$ and $i_L(0-)$. These will be the same just after time $t = 0$, $v_C(0+)$ and $i_L(0+)$ and will be your initial conditions.

Use normal circuit analysis to find your desired variable: $v_X(0)$ or $i_X(0)$

Also find: $\frac{d}{dt}v_X(0)$ or $\frac{d}{dt}i_X(0)$ The trick to finding these is to see that: $\frac{d}{dt}v_C(0) = \frac{i_C(0)}{C}$ and $\frac{d}{dt}i_L(0) = \frac{v_L(0)}{L}$

Find final conditions ("steady-state" or "forced" solution)

DC inputs: Inductors are shorts Capacitors are opens Solve by DC analysis $v_X(\infty)$ or $i_X(\infty)$

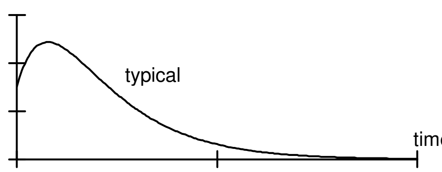
AC inputs: Solve by AC steady-state analysis using jω

X(t) may be replaced by $v_X(t)$, $i_X(t)$ or any desired variable in the equations below

<u>Overdamped</u> $b^2 - 4\cdot k > 0$ $\qquad s_1$ and s_2 are real and negative

$$X(t) = X(\infty) + B\cdot e^{s_1\cdot t} + D\cdot e^{s_2\cdot t}$$

$$X(0) = X(\infty) + B + D \qquad \frac{d}{dt}X(0) = B\cdot s_1 + D\cdot s_2$$ Solve simultaneously for B and D

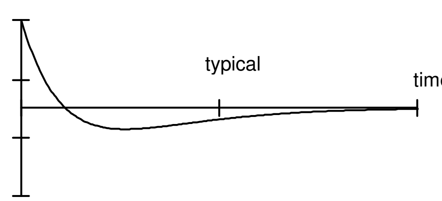


<u>Critically damped</u> $b^2 - 4\cdot k = 0$ $\qquad s_1 = s_2 = -\frac{b}{2} = s$ $\qquad s_1$ and s_2 are real, equal and negative

$$X(t) = X(\infty) + B\cdot e^{s\cdot t} + D\cdot t\cdot e^{s\cdot t}$$

$$X(0) = X(\infty) + B \qquad \frac{d}{dt}X(0) = B\cdot s + D \qquad \text{so.. } D = \frac{d}{dt}X(0) - B\cdot s$$

so.. $B = X(0) - X(\infty)$

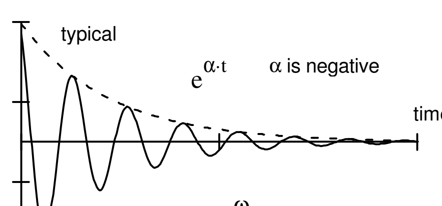


<u>Underdamped</u> $b^2 - 4\cdot k < 0$ $\qquad s_1 = \alpha + j\cdot\omega \qquad s_2 = \alpha - j\cdot\omega$ $\qquad \alpha$ is negative complex s_1 and s_2

$$X(t) = X(\infty) + e^{\alpha\cdot t}\cdot(B\cdot\cos(\omega\cdot t) + D\cdot\sin(\omega\cdot t))$$

$$X(0) = X(\infty) + B \qquad \frac{d}{dt}X(0) = B\cdot\alpha + D\cdot\omega \qquad \text{so.. } D = \frac{\frac{d}{dt}X(0) - B\cdot\alpha}{\omega}$$

so.. $B = X(0) - X(\infty)$



$f = \frac{\omega}{2\cdot\pi}$