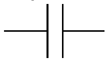
Notes, Second Order Transients

ECE 2210 A.Stolp 4/6/00, 10/24/05

			Laplace impedance	initial conditions	final conditions DC	final conditions AC
Capacitor 	$i_C = C \cdot \frac{d}{dt} v_C$	$v_C = \frac{1}{C} \cdot \int_{-\infty}^{t} i_C \, dt$	$\frac{1}{C \cdot s}$	short, or $v_C(0+) = v_C(0-)$	open	$\frac{1}{j \cdot \omega \cdot C}$
Inductor 	$v_L = L \cdot \frac{d}{dt} i_L$	$i_L = \frac{1}{L} \cdot \int_{-\infty}^{t} v_L \, dt$	$L \cdot s$	open, or $i_L(0+) = i_L(0-)$	short	$j \cdot \omega \cdot L$
Resistor 	$v_R = i_R \cdot R$	$i_R = \frac{v_R}{R}$	R	R	R	R

Characteristic equation

Use Laplace impedances, manipulate your circuit equation(s) into one equation of this form:

May be I_{in} or any forcing function / May be I_X or any desired variable

$$\left(a_1 \cdot s^2 + b_1 \cdot s + k_1\right) \cdot V_{in}(s) = \left(s^2 + b \cdot s + k\right) \cdot V_X(s)$$

Coefficients may be zero NO denominator s terms $s^2 + b \cdot s + k = 0$ = Characteristic equation

Differential equation

$$a_1 \cdot \frac{d^2}{dt^2} v_{in}(t) + b_1 \cdot v_{in}(t) + k_1 \cdot V_{in}(t) = \frac{d^2}{dt^2} v_X(t) + b \cdot \frac{d}{dt} v_X(t) + k \cdot v_X(t)$$

Transfer function

Rearrange circuit equation to: $H(s) = \frac{V_X(s)}{V_{in}(s)} = \frac{a_1 \cdot s^2 + b_1 \cdot s + k_1}{s^2 + b \cdot s + k}$ = transfer function

(May be I_X or any desired variable; May be I_{in} or any forcing function)

Complete solution

Solutions to the characteristic equation: $s_1 = -\frac{b}{2} + \frac{\sqrt{b^2 - 4 \cdot k}}{2}$ $\quad s_2 = -\frac{b}{2} - \frac{\sqrt{b^2 - 4 \cdot k}}{2}$

Find initial Conditions (v_C and/or i_L)

Find conditions of just before time t = 0, $v_C(0-)$ and $i_L(0-)$. These will be the same just after time t = 0, $v_C(0+)$ and $i_L(0+)$ and will be your initial conditions.

Use normal circuit analysis to find your desired variable: $v_X(0)$ or $i_X(0)$

Also find: $\frac{d}{dt} v_X(0)$ or $\frac{d}{dt} i_X(0)$ The trick to finding these is to see that: $\frac{d}{dt} v_C(0) = \frac{i_C(0)}{C}$ and $\frac{d}{dt} i_L(0) = \frac{v_L(0)}{L}$

Find final conditions ("steady-state" or "forced" solution)

DC inputs: Inductors are shorts Capacitors are opens Solve by DC analysis $v_X(\infty)$ or $i_X(\infty)$

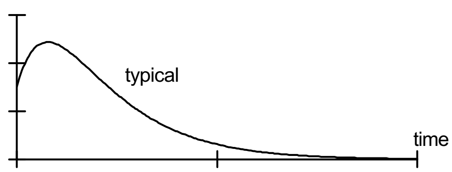
AC inputs: Solve by AC steady-state analysis using ω

$v_X(t)$ may be replaced by $i_X(t)$ or any desired variable in the equations below

Overdamped $b^2 - 4 \cdot k > 0$ s_1 and s_2 are real and negative

$$v_X(t) = v_X(\infty) + B \cdot e^{s_1 \cdot t} + D \cdot e^{s_2 \cdot t}$$

$v_X(0) = v_X(\infty) + B + D$ $\quad \frac{d}{dt} v_X(0) = B \cdot s_1 + D \cdot s_2$ Solve simultaneously for B and D

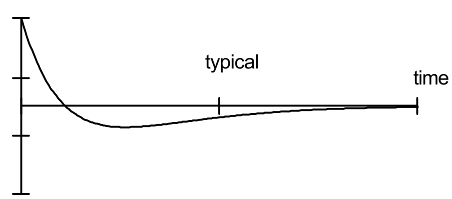


Critically damped $b^2 - 4 \cdot k = 0$ $\quad s_1 = s_2 = -\frac{b}{2} = s$ $\quad s_1$ and s_2 are real, equal and negative

$$v_X(t) = v_X(\infty) + B \cdot e^{s \cdot t} + D \cdot t \cdot e^{s \cdot t}$$

$v_X(0) = v_X(\infty) + B$ $\quad \frac{d}{dt} v_X(0) = B \cdot s + D$ $\quad$ so.. $D = \frac{d}{dt} v_X(0) - B \cdot s$

so.. $B = v_X(0) - v_X(\infty)$

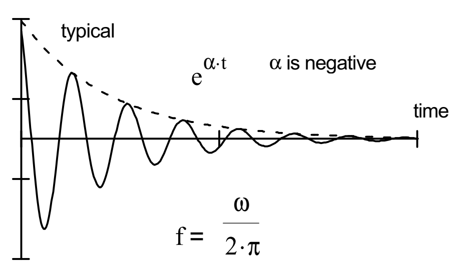


Underdamped $b^2 - 4 \cdot k < 0$ $\quad s_1 = \alpha + j \cdot \omega$ $\quad s_2 = \alpha - j \cdot \omega$ $\quad \alpha$ is negative

$$v_X(t) = v_X(\infty) + e^{\alpha \cdot t} \cdot (B \cdot \cos(\omega \cdot t) + D \cdot \sin(\omega \cdot t))$$

$v_X(0) = v_X(\infty) + B$ $\quad \frac{d}{dt} v_X(0) = B \cdot \alpha + D \cdot \omega$ so.. $D = \frac{\frac{d}{dt} v_X(0) - B \cdot \alpha}{\omega}$

so.. $B = v_X(0) - v_X(\infty)$



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